$$x^{\mu} = (x^0, x^1, x^2, x^3) \rightarrow dx^{\mu} = (dx^0, dx^1, dx^2, dx^3) = d\vec{X}$$

4차 위치벡터 $= (dx^0, d\vec{r})$

$$x_{\mu} = (x_0, x_1, x_2, x_3) = (x^0, -x^1, -x^2, -x^3)$$

$$x^{\mu} x_{\mu} = x^0 x_0 + x^1 x_1 + x^2 x_2 + x^3 x_3$$

$-x^1 \quad -x^2 \quad -x^3$

$$= (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = dx_{\nu} dx^{\nu}$$

$$ds^2 = dx_{\nu} dx^{\nu} = dx^{\nu} dx_{\nu}$$

$$\boxed{\beta = \frac{v}{c}}$$

$$\boxed{ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}}$$

$$g^{\mu\nu} x_{\nu} = x^{\mu}$$

$$\underset{1\times 4}{(x_0, x_1, x_2, x_3)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = x_0 - x_1 - x_2 - x_3$$

$4\times 1 \Rightarrow 1$

$$ds^2 = (dx^0, dx^1, dx^2, dx^3)(dx^0, -dx^1, -dx^2, -dx^3)$$

$$= (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$$

$$\boxed{dx^0 = c\,dt}$$

$$ds = dx^0 \sqrt{1 - \left(\frac{dx^1}{dx^0}\right)^2 - \left(\frac{dx^2}{dx^0}\right)^2 - \left(\frac{dx^3}{dx^0}\right)^2}$$

$$= dx^0 \sqrt{1 - \frac{1}{c^2}\left[\left(\frac{dx^1}{dt}\right)^2 + \left(\frac{dx^2}{dt}\right)^2 + \left(\frac{dx^3}{dt}\right)^2\right]}$$

$$\boxed{ds = dx^0 \sqrt{1 - \frac{v^2}{c^2}} = dx^0 \sqrt{1 - \beta^2}}$$

$$\vec{U} = \frac{d\vec{X}}{ds} = \frac{(dx^0, d\vec{r})}{dx^0 \sqrt{1 - \frac{v^2}{c^2}}}$$

4원 속도벡터

(dx^1, dx^2, dx^3)

$dx^0 = c\,dt$

$$\vec{U} = \left(\frac{1}{\sqrt{1-\beta^2}} \,\middle|\, \frac{d\vec{r}}{c\,dt\sqrt{1-\beta^2}}\right)$$

$$\boxed{\vec{U} = \left(\frac{1}{\sqrt{1-\beta^2}} \,\middle|\, \frac{v}{c\sqrt{1-\beta^2}}\right)} = ($$

$$P^{\mu} = mc\vec{U} = \left(\frac{mc}{\sqrt{1-\beta^2}} \,\middle|\, \frac{mv}{\sqrt{1-\beta^2}}\right)$$

$$\boxed{ds = dx^0 \sqrt{1 - \frac{v^2}{c^2}} = dx^0 \sqrt{1 - \beta^2}}$$

$$\vec{U} = \frac{d\vec{X}}{ds} = \frac{(dx^0, d\vec{r})}{dx^0 \sqrt{1 - \frac{v^2}{c^2}}}$$

4원 속도벡터

(dx^1, dx^2, dx^3)

$dx^0 = c\,dt$

$$\vec{U} = \left(\frac{1}{\sqrt{1-\beta^2}} \,\middle|\, \frac{d\vec{r}}{c\,dt\sqrt{1-\beta^2}}\right)$$

$$\boxed{\vec{U} = \left(\frac{1}{\sqrt{1-\beta^2}} \,\middle|\, \frac{v}{c\sqrt{1-\beta^2}}\right)}$$

$$= (U_0 \mid U_1, U_2, U_3)$$

$$P^{\mu} = mc\vec{U} = \left(\frac{mc}{\sqrt{1-\beta^2}} \,\middle|\, \frac{mv}{\sqrt{1-\beta^2}}\right)$$

$$W = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{x} = \int \frac{d\vec{p}}{dt} \cdot d\vec{x} = \int \frac{m}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \frac{dv}{dt} \cdot dx$$

$$\frac{dp}{dt} = \frac{d}{dt}\left(\frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}\right)$$

$$= m\frac{dv}{dt}\left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right) + mv\frac{d}{dt}\left[\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}\right]$$

$$-\frac{1}{2}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \frac{-2v}{c^2} \frac{dv}{dt}$$

$$= \left(\frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{mv^2}{c^2\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}\right) \frac{dv}{dt}$$

$$= \frac{m\left(1 - \frac{v^2}{c^2}\right)c^2 + mv^2}{c^2\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \frac{dv}{dt} = \frac{m}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \frac{dv}{dt}$$

$$W = \int \frac{mv}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} dv = \int \frac{mv}{x^{\frac{3}{2}}} \frac{-c^2}{2v} dx$$

$$\boxed{1 - \frac{v^2}{c^2} \equiv x}$$

$$\frac{-2v}{c^2} dv = dx$$

$$\boxed{\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$W = \frac{-mc^2}{2} \int_1^x x^{-\frac{3}{2}} dx$$

$$= \frac{-mc^2}{2} \frac{\left[x^{-\frac{3}{2}+1}\right]_{x=1}^{x}}{-\frac{3}{2}+1}$$

$$= mc^2\left(\frac{1}{\sqrt{x}}\right)_{x=0}^{x} = mc^2\left(\frac{1}{\sqrt{x}} - 1\right)$$

$$\boxed{W = mc^2\left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right) - mc^2} = K$$

운동에너지 $= K$

$$\boxed{\frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = K + mc^2}$$

$= E$

$$\boxed{E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mc^2 \qquad P = \gamma mv}$$

$$E^2 = \gamma^2 m^2 c^4$$

$\cdot d\vec{x} = \int \frac{m}{(1-\frac{v^2}{c^2})^{\frac{3}{2}}} \frac{dv}{dt} \cdot dx$

$W = \int \frac{mv}{(1-\frac{v^2}{c^2})^{\frac{3}{2}}} dv = \int \frac{mv}{x^{\frac{3}{2}}} \frac{-c^2}{2v} dx$

$\boxed{1-\frac{v^2}{c^2} \equiv x}$

$\frac{-2v}{c^2}dv = dx$

$\boxed{\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$

$\frac{d}{dt}\left[(1-\frac{v^2}{c^2})^{-\frac{1}{2}}\right]$

$-\frac{1}{2}(1-\frac{v^2}{c^2})^{-\frac{3}{2}} \frac{-2v}{c^2} \frac{dv}{dt}$

$)^{\frac{3}{2}}) \frac{dv}{dt}$

$W = \frac{-mc^2}{2} \int_1^x x^{-\frac{3}{2}} dx$

$= \frac{-mc^2}{2} \frac{\left[x^{-\frac{3}{2}+1}\right]_{x=1}^{x}}{-\frac{3}{2}+1}$

$\frac{m}{(1-\frac{v^2}{c^2})^{\frac{3}{2}}} \frac{dv}{dt}$

$= mc^2 \left(\frac{1}{\sqrt{x}}\right)_{x=0}^{x} = mc^2\left(\frac{1}{\sqrt{x}} - 1\right)$

$\boxed{W = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right) - mc^2} = K$

운동에너지 $= K$

$\boxed{\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = K + mc^2}$

$/\!/$ E

$$\boxed{\begin{gathered} E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma mc^2 \\ P = \gamma m v \end{gathered}}$$

$E^2 = \gamma^2 m^2 c^4$

$E^2 = \gamma^2 m^2 c^4 = m^2 c^4 \left(\frac{1}{1-\frac{v^2}{c^2}}\right)$

$= m^2 c^4 \frac{c^2}{c^2 - v^2} = m^2 c^2 \frac{c^2(c^2-v^2) + c^2 v^2}{c^2 - v^2}$

$= m^2c^2\left(c^2 + \frac{c^2v^2}{c^2-v^2}\right) = m^2c^2\left(c^2 + \frac{v^2}{1-\frac{v^2}{c^2}}\right)$

$= m^2c^4 + m^2c^2v^2\gamma^2$

$= (mc^2)^2 + (pc)^2$

$$\boxed{E = \sqrt{(mc^2)^2 + (pc)^2}}$$

$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{8\pi G}{c^4} T^{\mu\nu}$

$\boxed{T^{\mu\nu} = \rho_0 c^2 U^\mu U^\nu}$

energy density, energy flux

$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}$

momentum density, momentum flux, pressure

$\beta^i = \frac{v^i}{c}$

$T^{00} = \rho_0 c^2 U^0 U^0$ $\quad \beta^i\beta^j = \frac{v^i v^j}{c^2}$

$= \rho_0 c^2 \frac{1}{\sqrt{1-\beta^2}} \frac{1}{\sqrt{1-\beta^2}} = \frac{\rho_0}{1-\beta^2} c^2 = \rho c^2$

$T^{0i} = \rho_0 c^2 U^0 U^i = \frac{\rho_0}{1-\beta^2} c^2 \frac{v^i}{c} = \rho c^2 \beta^i$

$T^{ij} = \rho_0 c^2 U^i U^j$

$= \frac{\rho_0}{1-\beta^2} c^2 \beta^i \beta^j$

$= \rho c^2 \beta^i \beta^j$

$T^{\mu\nu} = \rho c^2 \begin{pmatrix} 1 & \beta^1 & \beta^2 & \beta^3 \\ \beta^1 & \beta^1\beta^1 & \beta^1\beta^2 & \beta^1\beta^3 \\ \beta^2 & \beta^2\beta^1 & \beta^2\beta^2 & \beta^2\beta^3 \\ \beta^3 & \beta^3\beta^1 & \beta^3\beta^2 & \beta^3\beta^3 \end{pmatrix}$

$\boxed{ds = dx^0\sqrt{1-\frac{v^2}{c^2}} = dx^0\sqrt{1-\beta^2}}$

$\vec{U} = \frac{d\vec{x}}{ds} = \frac{(dx^0, d\vec{r})}{dx^0\sqrt{1-\frac{v^2}{c^2}}}$ $\quad (dx^1, dx^2, dx^3)$ $\quad cdt$

4원 속도벡터

$\beta^i = \frac{v^i}{c}$

$\vec{U} = \left(\frac{1}{\sqrt{1-\beta^2}} \middle| \frac{d\vec{r}}{cdt\sqrt{1-\beta^2}}\right)$

$$\boxed{\vec{U} = \left(\frac{1}{\sqrt{1-\beta^2}} \middle| \frac{v}{c\sqrt{1-\beta^2}}\right)}$$

$P^\mu = mc\vec{U} = \left(\overbrace{\frac{mc}{\sqrt{1-\beta^2}}}^{=p^0} \middle| \overbrace{\frac{mv}{\sqrt{1-\beta^2}}}^{=p^i}\right)$

$x^3)$ $\quad cdt$ $\quad \beta^2]$

$\frac{v}{c} \ll 1 \Rightarrow \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \simeq \left(1+\frac{1}{2}\frac{v^2}{c^2}\right)$

$W = mc^2\left(1+\frac{1}{2}\frac{v^2}{c^2}\right) - mc^2 = \frac{1}{2}mv^2$

$E = K.E + P.E. = \frac{1}{2}mv^2 + V(\vec{r})$

$E = \frac{1}{2}mv^2 + V = \frac{(mv)^2}{2m} + V(\vec{r})$

$H = \frac{p^2}{2m} + V(r)$

$$\boxed{H \to i\hbar\frac{\partial}{\partial t} \ (E) \qquad \vec{p} \to -i\hbar\nabla = -i\hbar\frac{\partial}{\partial x}}$$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi + V\psi}$$

슈뢰딩거 방정식

$\psi^* \times \square$

$\psi \times \square^*$

$\frac{\partial}{\partial t}(\underbrace{\psi^*\psi}_{\geq 0}) = \frac{\partial\rho}{\partial t}$ $\quad \psi^*\psi = \rho$

$\boxed{\frac{\partial\rho}{\partial t} + \nabla\cdot\vec{J} = 0}$

$E^2 = (mc^2)^2 + (pc)^2$

$$\boxed{\left(i\hbar\frac{\partial}{\partial t}\right)^2\psi = (mc^2)^2\psi + (-i\hbar c\nabla)^2\psi}$$

클라인-고든 eq. K-G

$\frac{\partial}{\partial t}\left(\underbrace{\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}}_{\rho}\right)$ $\to \leq 0$

$E = \pm\sqrt{(pc)^2 + (mc^2)^2}$ $\quad p \to -i\hbar\nabla$

$i\hbar\frac{\partial\psi}{\partial t} = \left(-i\hbar c\,\vec{\alpha}\cdot\vec{\nabla} + (mc^2)\beta\right)$

$i\hbar\frac{\partial\psi}{\partial t} + i\hbar c\,\vec{\alpha}\cdot\vec{\nabla}\psi - mc^2\beta\psi$

$i\hbar\left(\frac{\partial}{\partial t} + c\vec{\alpha}\cdot\nabla\right)\psi - mc^2\beta\psi$

$\times\frac{\beta}{c}$

$i\hbar\left(\beta\frac{\partial}{c\partial t} + \beta\vec{\alpha}\frac{\partial}{\partial x^i}\right)\psi - mc$

$i\hbar\left(\beta\frac{\partial}{\partial x^0} + \beta\vec{\alpha}\frac{\partial}{\partial x^i}\right)\psi - mc\beta^2$

$\left(\ \right)^{-\frac{1}{2}} \simeq \left(1+\frac{1}{2}\frac{v^2}{c^2}\right)$

$\frac{v^2}{c^2}\Big) - mc^2 = \frac{1}{2}mv^2$

$E. = \frac{1}{2}mv^2 + V(\vec{r})$

$-V = \frac{(mv)^2}{2m} + V(\vec{r})$

$+ V(r)$

$\boxed{\vec{p} \rightarrow -i\hbar\nabla = -i\hbar\frac{\partial}{\partial x}}$

$\boxed{\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi}$

$\frac{}{}(\underbrace{\psi^*\psi}_{\geq 0}) = \frac{\partial\rho}{\partial t}$

$\boxed{\frac{\partial\rho}{\partial t} + \nabla\cdot\vec{J} = 0}$

$E^2 = (mc^2)^2 + (pc)^2$

$\boxed{\left(i\hbar\frac{\partial}{\partial t}\right)^2\psi = (mc^2)^2\psi + (-i\hbar c\nabla)^2\psi}$

K-G eq.

$\frac{\partial}{\partial t}\Big(\underbrace{\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}}_{\rho}\Big) \rightarrow \leq 0$

$E = \pm\sqrt{(pc)^2 + (mc^2)^2}$

$i\hbar\frac{\partial\psi}{\partial t} = \left(-i\hbar c\,\vec{\alpha}\cdot\vec{\nabla} + (mc^2)\beta\right)\psi$

$i\hbar\frac{\partial\psi}{\partial t} + i\hbar c\,\vec{\alpha}\cdot\vec{\nabla}\psi - mc^2\beta\psi = 0$

$i\hbar\left(\frac{\partial}{\partial t} + c\vec{\alpha}\cdot\nabla\right)\psi - mc^2\beta\psi = 0$

$\times\frac{\beta}{c}$ $\quad i\hbar\left(\beta\frac{\partial}{c\,dt} + \beta\vec{\alpha}\frac{\partial}{\partial x^i}\right)\psi - mc\beta^2\psi = 0$

$i\hbar\left(\beta\frac{\partial}{dx^0} + \beta\vec{\alpha}\frac{\partial}{\partial x^i}\right)\psi - mc\beta^2\psi = 0$

$\boxed{\gamma^i = \beta\alpha^i}$ $\quad \beta = \gamma^0 \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$

$i\hbar\left(\gamma^0\frac{\partial}{\partial x^0} + \gamma^i\frac{\partial}{\partial x^i}\right)\psi - mc\psi = 0$

$\left(i\hbar\gamma^\mu\frac{\partial}{\partial x^\mu} - mc\right)\psi = 0$

$\boxed{(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0}$

$i\hbar\partial_\mu = i\hbar\frac{\partial}{\partial x^\mu} = P_\mu$

$\boxed{(\gamma^\mu P_\mu - mc)\psi = 0}$

Dirac eq

$\gamma^i = \beta\alpha^i \quad \boxed{\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$

$\alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}$

$\boxed{T^{\mu\nu} = \rho_0 c^2 U^\mu U^\nu}$

$\beta^2 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} = 1$

$(\gamma^0 P_0 + \gamma^i P_i - mc)\psi = 0$

$(\gamma^0 p^0 - \gamma^i p^i - mc)\psi = 0$

$\gamma^0 = \beta \quad p^0 = \frac{mc}{\sqrt{1-\beta^2}} = \gamma mc \quad E = \gamma mc^2$

$\gamma^0 p^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}\frac{E}{c} = \begin{pmatrix} \frac{E}{c} & 0 \\ 0 & -\frac{E}{c} \end{pmatrix}$

$\gamma^i = \beta\alpha^i = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}\begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$

$\gamma^i p^i = \vec{\gamma}\cdot\vec{p}$

$I_-|p\rangle = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$I_+|n\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$x^\mu = (x^0, x^1, x^2, x^3) \rightarrow dx^\mu = (dx^0, dx^1, dx^2, dx^3) = d\vec{X}$

4차 위치벡터 $= (dx^0, d\vec{r})$

$x_\mu = (x_0, x_1, x_2, x_3) = (x^0, -x^1, -x^2, -x^3)$

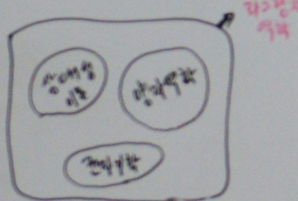
$x^\mu x_\mu = x^0x_0 + x^1x_1 + x^2x_2 + x^3x_3$

$= (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$

$\boxed{ds^2 = g_{\mu\nu}dx^\mu dx^\nu}$

$g^{\mu\nu}x_\nu = x^\mu$

$(x_0, x_1, x_2, x_3)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = x^0 - x^1 - x^2 - x^3$



뉴턴 역학 $\quad$ 라그랑지 역학

$\vec{F}, \vec{P}$ $\quad$ 운동에너지, 위치에너지

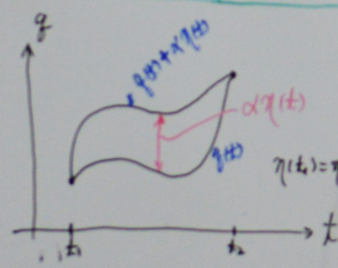
힘 $\quad$ 에너지

에너지 · 시간 ⇒ 작용

최소작용 원칙

= 해밀톤 원칙

$\boxed{ds = dx^0\sqrt{1-\frac{v^2}{c^2}} = dx^0\sqrt{1-\beta^2}}$



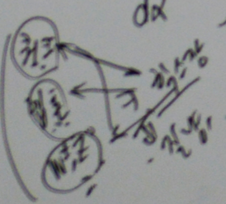
$\eta(t_1) = \eta(t_2) = 0$

$q(\alpha, t) = q(t) + \alpha\eta(t) \rightarrow \dot{q} = \dot{q} + \alpha\dot{\eta}$

$\frac{\partial\dot{q}}{\partial\alpha} = \dot{\eta}$

$S = \int_{t_1}^{t_2} L(q, \dot{q})\,dt$

$\frac{ds}{d\alpha} = \int\left(\frac{\partial L}{\partial q}\frac{dq}{d\alpha} + \frac{\partial L}{\partial\dot{q}}\frac{d\dot{q}}{d\alpha}\right)dt = 0$



$S = \int \mathcal{L}(\phi, \partial_\mu\phi)\,d^4x \quad \mathcal{L}(\phi,$

$L(q,$

$ds = dx^0\sqrt{1-\frac{v^2}{c^2}} = dx^0\sqrt{1-\beta^2}$
$d\vec{X} = (dx^0, d\vec{r})$
$\eta(t_1) = \eta(t_2) = 0$
$q(\alpha, t) = q(t) + \alpha\eta(t) \rightarrow \dot{q} = \dot{q} + \alpha\dot{\eta}$
$\frac{\partial q}{\partial \alpha} = \eta$
$S = \int_{t_1}^{t_2} L(q, \dot{q})\, dt$
$\frac{dS}{d\alpha} = \int \left(\frac{\partial L}{\partial q}\frac{dq}{d\alpha} + \frac{\partial L}{\partial \dot{q}}\frac{d\dot{q}}{d\alpha}\right) dt = 0$
$S = \int \mathcal{L}(\phi, \partial_\mu\phi)\, d^4x$
$\mathcal{L}(\phi, \partial_\mu\phi)$
$L(q, \dot{q})$
$\int \frac{\partial L}{\partial \dot{q}}\frac{d\dot{q}}{d\alpha} dt$
$(xy)' = x'y + xy'$
$\int (xy)' = \int x'y + \int xy'$
$\int xy' = xy - \int x'y$
$\int \frac{\partial L}{\partial \dot{q}}\frac{d\dot{q}}{d\alpha} dt = \left[\frac{\partial L}{\partial \dot{q}}\eta\right]_{t_1}^{t_2} - \int \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\eta\, dt$
$\frac{dS}{d\alpha} = \int \eta\left[\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\right] dt = 0$
$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$
$[L_x, L_y] = i\epsilon_{xyz} L_z$
$\dot{q} = \frac{\partial H}{\partial p}$
$\dot{p} = -\frac{\partial H}{\partial q}$
해밀토니안
$[q, H] = \dot{q}$
$[p, H] = \dot{p}$
$\frac{\partial L}{\partial q} = 0 \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$
공간대칭
$\frac{\partial L}{\partial \dot{q}} \equiv p$ = 상수
$\frac{\partial L}{\partial t} = 0$
$\frac{\partial}{\partial t}L(q, \dot{q}) = \frac{\partial L}{\partial q}\frac{\partial q}{\partial t} + \frac{\partial L}{\partial \dot{q}}\frac{\partial \dot{q}}{\partial t} = \frac{\partial L}{\partial q}\dot{q} + \frac{\partial L}{\partial \dot{q}}\ddot{q}$
$= \dot{q}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\partial L}{\partial \dot{q}}\ddot{q} = \frac{d}{dt}\left(\dot{q}\frac{\partial L}{\partial \dot{q}}\right)$
$\frac{d}{dt}\left(\dot{q}\frac{\partial L}{\partial \dot{q}} - L\right) = 0$
$\dot{q}\frac{\partial L}{\partial \dot{q}} - L \equiv H$
$L = T - V$
$= \frac{1}{2}mv^2 - \frac{1}{2}kx^2$
$\frac{\partial L}{\partial q} = -kx \quad \frac{\partial L}{\partial \dot{q}} = mv$
$-kx - \frac{d}{dt}(mv) = 0$
$ma = -kx$
$F = -kx$
$\frac{E - mc^2}{c} U_A - \vec{\sigma}\cdot\vec{p}\, U_B = 0$
$\vec{\sigma}\cdot\vec{p}\, U_A - \frac{E + mc^2}{c} U_B = 0$
$U_A = \frac{c}{E - mc^2}(\vec{\sigma}\cdot\vec{p})U_B$
$U_B = \frac{c}{E + mc^2}(\vec{\sigma}\cdot\vec{p})U_A$
$U_B = \frac{c}{E + mc^2}\begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{c}{E + mc^2}$
대칭 ⟷ 보존
변화 없는 = 상수
골드스톤 힉스보손
$S = 0$
$m = 0$
대칭붕괴
힉스
H^+ H^0 H^- H^0
W^+ Z^0 W^-
$\Rightarrow Z^0, W^\pm, \gamma$
$U(1)$
antimatter
spin-up $\Rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$I_-|p\rangle = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} =$
$I_+|n\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} =$