$$x^\mu \quad A^\mu \quad j^\mu \quad P^\mu \quad D^\mu \quad \gamma^\mu \Rightarrow \mu = 0, 1, 2, 3$$

$$= t, x, y, z$$

four vector

$$x^\mu = (x^0\ x^1\ x^2\ x^3) = (t, \vec{x})$$

$$A^\mu = (A^0, \vec{A}) = (\phi, \vec{A})$$

$$j^\mu = (j^0, \vec{j}^i) = (\rho, \vec{J})$$

$$P^\mu = (P^0, \vec{P}) = (E, \vec{p}) \rightarrow E^2 = (pc)^2 + (mc^2)^2$$

$$D^\mu = (D^0, \vec{D}) \qquad \gamma^\mu = (\gamma^0, \vec{\gamma}^i)$$

Dirac 입자 local U(1) gauge 변환

$$\psi' \longrightarrow e^{iq\theta(x)}\psi \quad \bar{\psi}' \rightarrow e^{-iq\theta(x)}\bar{\psi}$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \qquad \partial_\mu e^{iq\theta(x)} = e^{iq\theta} \cdot iq\,\partial_\mu\theta$$

$$\mathcal{L}' = \bar{\psi}'(i\gamma^\mu \partial_\mu - m)\psi'$$

$$= e^{-iq\theta}\bar{\psi}(i\gamma^\mu\partial_\mu - m)e^{iq\theta}\psi$$

$$= e^{-iq\theta}\bar{\psi}i\gamma^\mu\partial_\mu(e^{iq\theta}\psi) - e^{-iq\theta}\bar{\psi}me^{iq\theta}\psi$$

$$= e^{-iq\theta}\bar{\psi}i\gamma^\mu\psi e^{iq\theta}iq\,\partial_\mu\theta + e^{-iq\theta}\bar{\psi}i\gamma^\mu e^{iq\theta}\partial_\mu\psi - \bar{\psi}m\psi$$

$$= \bar{\psi}i\gamma^\mu\psi\, iq\,\partial_\mu\theta + \bar{\psi}i\gamma^\mu\partial_\mu\psi - \bar{\psi}m\psi \qquad \mathcal{L}' = \mathcal{L}$$

$$= \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \bar{\psi}i\gamma^\mu\psi\, iq\,\partial_\mu\theta = \mathcal{L} + \bar{\psi}i\gamma^\mu\psi\, iq\,\partial_\mu\theta$$

gauge 입자 local U(1) gauge 변환

$$B_\mu \to B_\mu' = B_\mu - \frac{1}{g_1}\partial_\mu \chi$$

$$\mathcal{L} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - j^\mu B_\mu + \frac{1}{2}m^2 B^\mu B_\mu$$

$$\mathcal{L}' = -\frac{1}{4}B'^{\mu\nu}B'_{\mu\nu} - j^\mu B'_\mu \qquad B^{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$B'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu = \partial_\mu\left(B_\nu - \frac{1}{g_1}\partial_\nu \chi\right) - \partial_\nu\left(B_\mu - \frac{1}{g_1}\partial_\mu\chi\right)$$

$$= \partial_\mu B_\nu - \partial_\nu B_\mu - \frac{1}{g_1}\partial_\mu\partial_\nu\chi + \frac{1}{g_1}\partial_\nu\partial_\mu\chi = \partial_\mu B_\nu - \partial_\nu B_\mu = B_{\mu\nu}$$

$$j^\mu B'_\mu = j^\mu\left(B_\mu - \frac{1}{g_1}\partial_\mu\chi\right) = j^\mu B_\mu - j^\mu\frac{1}{g_1}\partial_\mu\chi$$

$$\bar{\psi} i\gamma^\mu \psi\, i a\, \partial_\mu\theta + j^\mu \frac{1}{g_1}\partial_\mu \chi = 0$$

$$j^\mu = a g_1 \bar{\psi}\gamma^\mu\psi$$

Dirac + gauge ⇒ U(1) gauge 변환

$$\mathcal{L} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - j^\mu B_\mu$$

$$= \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \overset{i\cdot i}{-} a g_1 B_\mu \bar{\Psi}\gamma^\mu\psi$$

$$= \bar{\Psi}\left[i\gamma^\mu(\partial_\mu + i a g_1 B_\mu) - m\right]\psi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$

$$= \bar{\Psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$

$$D_\mu = \partial_\mu + i \underset{\text{Dirac 입자}}{a} \underset{\text{gauge 입자}}{g_1} B_\mu$$

$$D_\mu = \partial_\mu - i g_1 \frac{Y}{2} B_\mu - i g_2 \frac{\tau^i}{2} W^i_\mu - i g_3 \frac{\lambda^a}{2} G^a_\mu$$

$$P^\mu = (P^0 \ \vec{P}) = (E, \vec{P})$$

$$E = i\partial_0 \qquad \vec{P} = -i\nabla \quad \hbar = 1$$

$$E^2 = (pc)^2 + (mc^2)^2 \quad c = 1$$

$$E^2 = p^2 + m^2 \qquad -\partial_0\partial_0 = -\nabla^2 + m^2$$

$$\partial_0 = \partial^0 \qquad \underbrace{-\partial_0\partial^0 + \nabla^2}_{} = m^2 \qquad \nabla^2 = \vec{\nabla}\cdot\vec{\nabla}$$

$$\hookrightarrow \partial_\mu\partial^\mu = \partial_0\partial^0 + \partial_1\partial^1 \ldots$$

$$= \partial_0\partial^0 - \nabla^2$$

$$-\partial_\mu\partial^\mu = m^2$$

$$(\partial_\mu\partial^\mu + m^2)\phi = 0$$

K-G

$$(D^\mu \phi)^\dagger D_\mu \phi - V(|\phi|^2) \qquad \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h+v \end{pmatrix}$$

$$= \frac{1}{2}\partial^\mu h\, \partial_\mu h + \frac{1}{8}(g_1^2+g_2)^2 Z^\mu Z_\mu (h+v)^2 + \frac{1}{4} g_2^2\, W^{-\mu} W^+_\mu (h+v)^2$$

$$- \frac{\lambda}{4}(h^4 + 4h^3 v + 4h^2 v^2) \qquad m_h = \sqrt{2\lambda v^2}$$

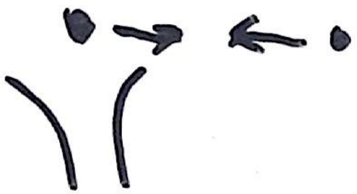
$$x^\mu = (x^0\ x^1\ x^2\ x^3) = (t,\ x,\ y,\ z) = (t,\ \vec{x})$$

$$x_\mu = (x_0\ x_1\ x_2\ x_3) = (x^0\ -x^1\ -x^2\ -x^3) \qquad \partial^\mu \partial_\mu = \partial_\mu \partial^\mu$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial x^0}\ \overset{\frac{\partial}{\partial x}}{\frac{\partial}{\partial x^1}}\ \overset{\frac{\partial}{\partial y}}{\frac{\partial}{\partial x^2}}\ \overset{\frac{\partial}{\partial z}}{\frac{\partial}{\partial x^3}}\right) = (\partial^0,\ \vec{\nabla}) \qquad \partial^\mu = \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial x_0}\ \frac{\partial}{\partial x_1}\ \frac{\partial}{\partial x_2}\ \frac{\partial}{\partial x_3}\right) = (\partial_0,\ -\vec{\nabla})$$

boson ⇒
- scalar boson ($S=0$)
  - $S=0$, $m=0$ → Goldston boson
  - $S=0$, $m\neq 0$ → Higgs boson
- gauge boson ($S=1$)
  - $S=1$, $m=0$ → γ
  - $S=1$, $m\neq 0$ → W^+, W^-, Z^0

Dirac 입자 ⇒ $S=\frac{1}{2}$



$\mathrm{fm} \rightarrow 10^{-15}\,\mathrm{m}$

$$V(r) = \frac{ke^2}{r} = \frac{1.44\ \mathrm{MeV\cdot fm}}{10\ \mathrm{fm}} = 144\ \mathrm{KeV}$$

$\rightarrow 70\ \mathrm{KeV}$

$$\langle K \rangle = \frac{3}{2} k_B T$$

$$T = \frac{2}{3}\frac{\langle K \rangle}{k_B} = \frac{2}{3}\frac{7\times10^{4}\ \mathrm{eV}}{8.6\times10^{-5}\ \mathrm{eV/K}}$$

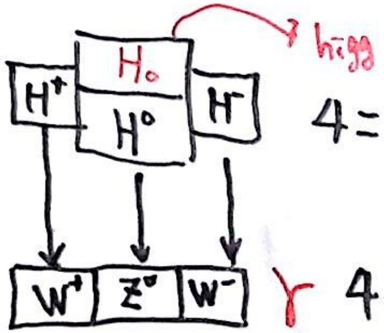
$$\simeq 5\times10^{8}\ \mathrm{K} = 5억\ \mathrm{K}$$

$$\frac{10^{-15}\ \mathrm{m}}{3\times10^{8}\ \mathrm{m/sec}} \simeq 10^{-23}\ \mathrm{sec}$$

$$W_\mu^3 = \underset{\sim}{Z_\mu} \cos\theta_W + \underset{\sim}{A_\mu} \sin\theta_W$$

$\sin^2\theta_W \simeq 0.23$

$$B_\mu = -Z_\mu \sin\theta_W + A_\mu \cos\theta_W$$



$4 = 3 + 1$

γ $4 = 3 + 1$

약력 전자기

$$SU_C(3) \times \underbrace{SU_L(2) \times U_Y(1)}$$

$\hookrightarrow W^\pm, Z_0$ $\hookrightarrow \gamma$

lepton 과 유카와장의 결합 ⇒ $\bar{\Psi}\phi\Psi$ $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h+v \end{pmatrix}$ $\phi^{\dagger} = \frac{1}{\sqrt{2}}(0 \quad h+v)$

$-g_l \bar{l}_L \phi e_R - g_l \bar{e}_R \phi^{\dagger} l_L$ $l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ $\bar{l}_L = (\bar{\nu}_L \ \bar{e}_L)$ $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \gamma^0$

$\beta\alpha^i = \gamma^i = \begin{pmatrix} 0 & \alpha^i \\ -\alpha^i & 0 \end{pmatrix}$

$P_R P_L = 1$

$= -g_l (\bar{\nu}_L \ \bar{e}_L) \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h+v \end{pmatrix} e_R - g_l \bar{e}_R \frac{1}{\sqrt{2}}(0 \quad h+v)\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ $P_L = \frac{1-\gamma^5}{2}$ $P_R = \frac{1+\gamma^5}{2}$ $P_L P_R = 0$

$1\times 2 \quad 2\times 1 \rightarrow 1\times 1$

$= \frac{-1}{\sqrt{2}} g_l \bar{e}_L (h+v) e_R - \frac{1}{\sqrt{2}} g_l \bar{e}_R (0 \quad h+v) e_L$ $P_L e = e_L$ $\bar{e} P_L = \bar{e}_R P_R$

$= \frac{-1}{\sqrt{2}} g_l (h+v)(\bar{e}_L e_R + \bar{e}_R e_L) = \frac{-1}{\sqrt{2}} g_l (h+v) \bar{e} e$ $m_e = \frac{1}{\sqrt{2}} g_l v$ $m_d = \frac{1}{\sqrt{2}} g_d v$

$e = e_L + e_R$ $\bar{e}e = (\bar{e}_L + \bar{e}_R)(e_L + e_R)$ $m_u = \frac{1}{\sqrt{2}} g_u v$

$= \bar{e}_L \overset{=0}{e_L} + \bar{e}_L e_R + \bar{e}_R e_L + \bar{e}_R \overset{=0}{e_R}$

$= \bar{e} \underbrace{P_R P_L}_{0} e + \bar{e} P_R P_R e + \bar{e} P_L P_L e + \bar{e} \underbrace{P_L P_R}_{0} e$

$= -\frac{1}{\sqrt{2}} g_l h \bar{e} e - \frac{1}{\sqrt{2}} g_l v \bar{e} e$

$$-g_d \bar{q}_L \phi d_R - g_d \bar{d}_R \phi^\dagger q_L \qquad q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$= -\frac{1}{\sqrt{2}} g_d (\bar{u}_L \ \bar{d}_L) \begin{pmatrix} 0 \\ h+v \end{pmatrix} d_R - \frac{1}{\sqrt{2}} g_d \bar{d}_R (0 \ \ h+v) \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$= -\frac{1}{\sqrt{2}} g_d \bar{d}_L (h+v) d_R - \frac{1}{\sqrt{2}} g_d \bar{d}_R (h+v) d_L$$

$$= \frac{-1}{\sqrt{2}} g_d (h+v) (\bar{d}_L d_R + \bar{d}_R d_L)$$

$$= -\frac{1}{\sqrt{2}} g_d (h+v) \bar{d} d = -\frac{1}{\sqrt{2}} g_d h \bar{d} d - \frac{1}{\sqrt{2}} g_d \underline{v} \bar{d} d$$

$m_{W^\pm} = \frac{1}{2} v g_2$ (80 GeV) $m_h = 125$ GeV

$m_Z = \frac{1}{2} v \sqrt{g_1^2 + g_2^2} = 91$ GeV

$\gamma^\mu P_L = P_R \gamma^\mu$

$\bar{\psi}\psi = \psi^\dagger \gamma^0 \psi$ → scalar

$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$ 4×1

$\bar{\psi}\gamma^0\psi \to \bar{\psi}\gamma^\mu\psi$ → zero.

$\bar{\psi} = \psi^\dagger \gamma^0$

$\bar{\psi}\gamma^5\psi$ → psedo-scalar

$\psi^\dagger = (\psi_1^* \; \psi_2^* \; \psi_3^* \; \psi_4^*)$

$\bar{\psi}\gamma^\mu\psi$ → 4-vector, 4

$\bar{\psi}\gamma^\mu\gamma^5\psi$ → psedo-4-vector, 4

$\bar{\psi}\sigma^{\mu\nu}\psi$ → tensor, 6개

$\bar{\psi} = \psi^\dagger\gamma^0 = (\psi_1^* \psi_2^* \psi_3^* \psi_4^*)\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&-1&0 \\ 0&0&0&-1 \end{pmatrix}$

$= (\psi_1^* \; \psi_2^* \; -\psi_3^* \; -\psi_4^*)$

1×4 4×1 → 1×1 →

$\bar{\psi}\psi = |\psi_1|^2 + |\psi_2|^2 - |\psi_3|^2 - |\psi_4|^2$

$\bar{\psi}_L\psi_L = 0 \quad \bar{\psi}_L\psi_R$

$\bar{\psi}\gamma^\mu\psi = (\bar{\psi}_L + \bar{\psi}_R)\gamma^\mu(\psi_L + \psi_R)$

$= (\bar{\psi}_L + \bar{\psi}_R)(\gamma^\mu\psi_L + \gamma^\mu\psi_R)$

$= \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_L\gamma^\mu\overset{=0}{\psi_R} + \bar{\psi}_R\gamma^\mu\overset{=0}{\psi_L} + \bar{\psi}_R\gamma^\mu\psi_R$

$\underbrace{\bar{\psi}P_R\gamma^\mu P_R\psi}_{\bar{\psi}\underbrace{P_R P_L}_{0}\gamma^\mu\psi} + \underbrace{\bar{\psi}P_L\gamma^\mu P_L\psi}_{0}$

$\bar{\psi}\gamma^\mu\psi = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R$ → helicity is preserved.

$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \to \bar{\psi}\gamma^\mu\partial_\mu\psi$

$A = A^T$ Symmetric matrix A

$A = A^\dagger$ Hermitian " A

$A^{-1} = A^\dagger$ Unitary " A

$\bar{\psi}_L\gamma^\mu\psi_L = \frac{1-\gamma^5}{2}\gamma^\mu\frac{1+\gamma^5}{2}$ ⇒ A-V interaction

$AA^{-1} = AA^\dagger = I$

Dirac 입자 ⇒ $S = \frac{1}{2}$

A_μ

$\nabla \cdot E = 0 \qquad \nabla \cdot B = 0$

$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \times B = \frac{\partial E}{\partial t}$

$\vec{E} = -\nabla \overset{A^0}{\phi} - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \nabla \times \vec{A} \Rightarrow \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu \underline{\underline{A_\mu}}$

$j^\nu = \partial_\mu F^{\mu\nu}$

$\mathcal{L} = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - j^\mu B_\mu + \frac{1}{2} m^2 B^\mu B_\mu$

$F_{\mu\nu} = \partial_\mu \underline{A_\nu} - \partial_\nu \underline{A_\mu}$

$j^\nu = \partial_\mu B^{\mu\nu} + \underline{m^2 B^\mu B_\mu}$

$W_\mu^i \rightarrow \mathcal{L} = -\frac{1}{4} W^{\mu\nu} W_{\mu\nu} - j^\mu W_\mu + \frac{1}{2} m^2 W^\mu W_\mu$

$\underbrace{W_\mu^1, W_\mu^2}, W_\mu^3$

$\Downarrow \qquad \Downarrow B_\mu$

$W^\pm \qquad \Downarrow$

Z^0, γ

$j^\nu = \partial_\mu W^{\mu\nu} + m^2 W^\mu W_\mu + g_2 (W_\mu \times W^{\mu\nu})$

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$F_{\mu\nu} = -F_{\nu\mu}$

$F^{\mu\nu} = F_{\mu\nu}$