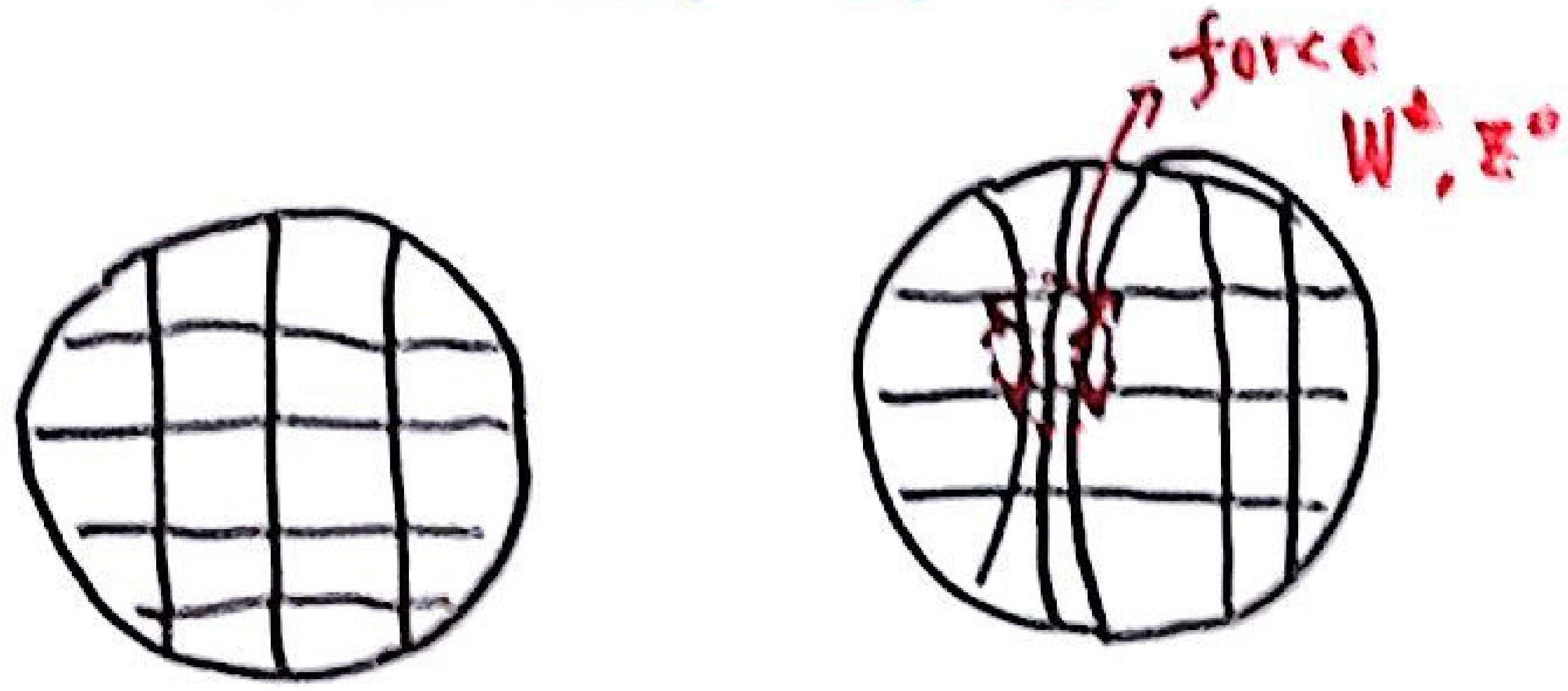
		Fermion			Boson	
quark	$\frac{2}{3}$	u	c	t	γ	→ 전자기 상호작용 $U_Y(1)$
	$-\frac{1}{3}$	d	s	b	W^+, W^-, Z^0	→ 약력 $SU_L(2)$
lepton	-1	e	μ	τ	g	→ 강력 $SU_c(3)$
	0	ν_e	ν_μ	ν_τ		
Higgs						

$$SU_c(3) \times SU_L(2) \times U_Y(1)$$

$\hookrightarrow g$ strong, $\hookrightarrow W^\pm, Z^0$ Weak, $\hookrightarrow \gamma$ EM

⇒ Global, Local

particle ⇒ m, c, s



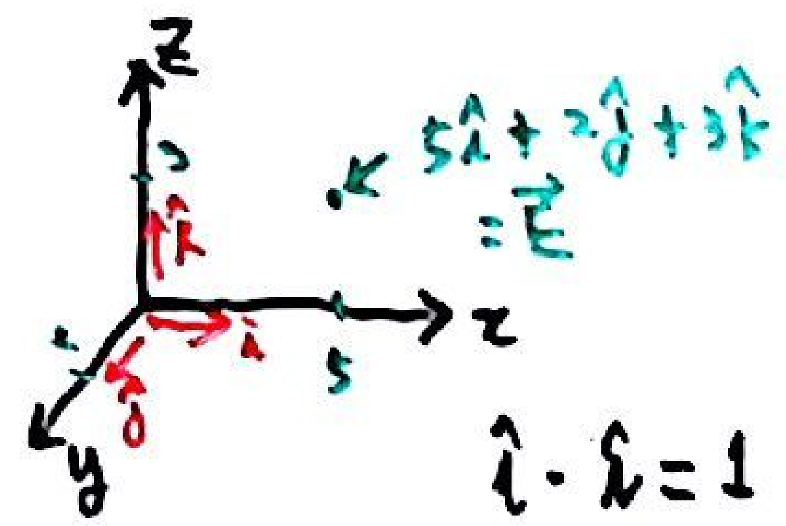
local gauge symmetry

양. → 수 ⇒ Scalar, vector, tensor

orthogonal, Complete

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \cdot \vec{H} = 0 \qquad \nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$



$$\nabla \cdot \vec{E} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right)$$

$\hat{i}\cdot\hat{i} = 1$

$\hat{i}\cdot\hat{j} = 0$ ↳ cos

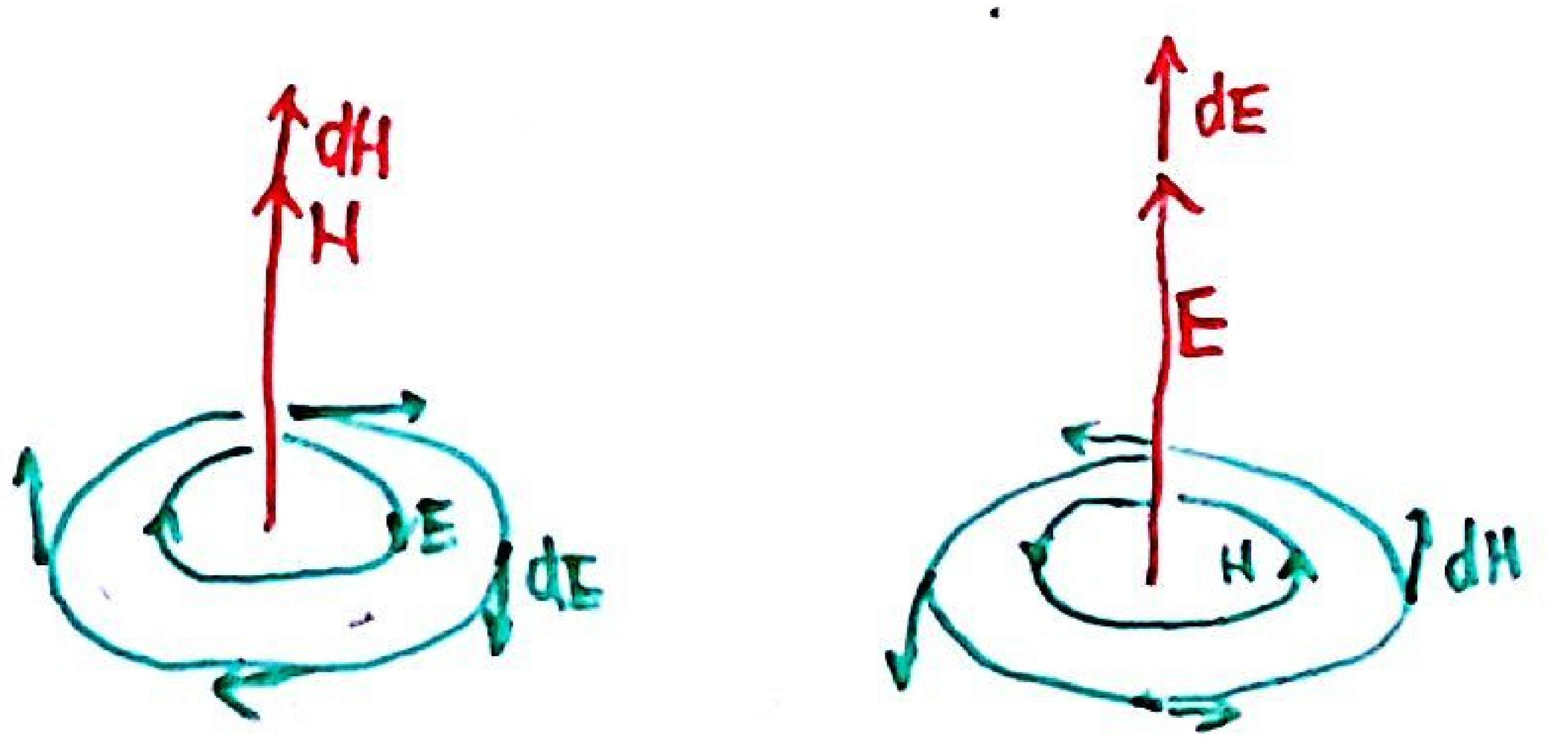
$\nabla \qquad \cdot \qquad E$

$$= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{i} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{j}$$

$$+ \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{k}$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \qquad \nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$



$$\nabla \times (\nabla \times \vec{E}) = \nabla \times \left(-\mu_0 \frac{\partial \vec{H}}{\partial t}\right) = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \vec{H})$$

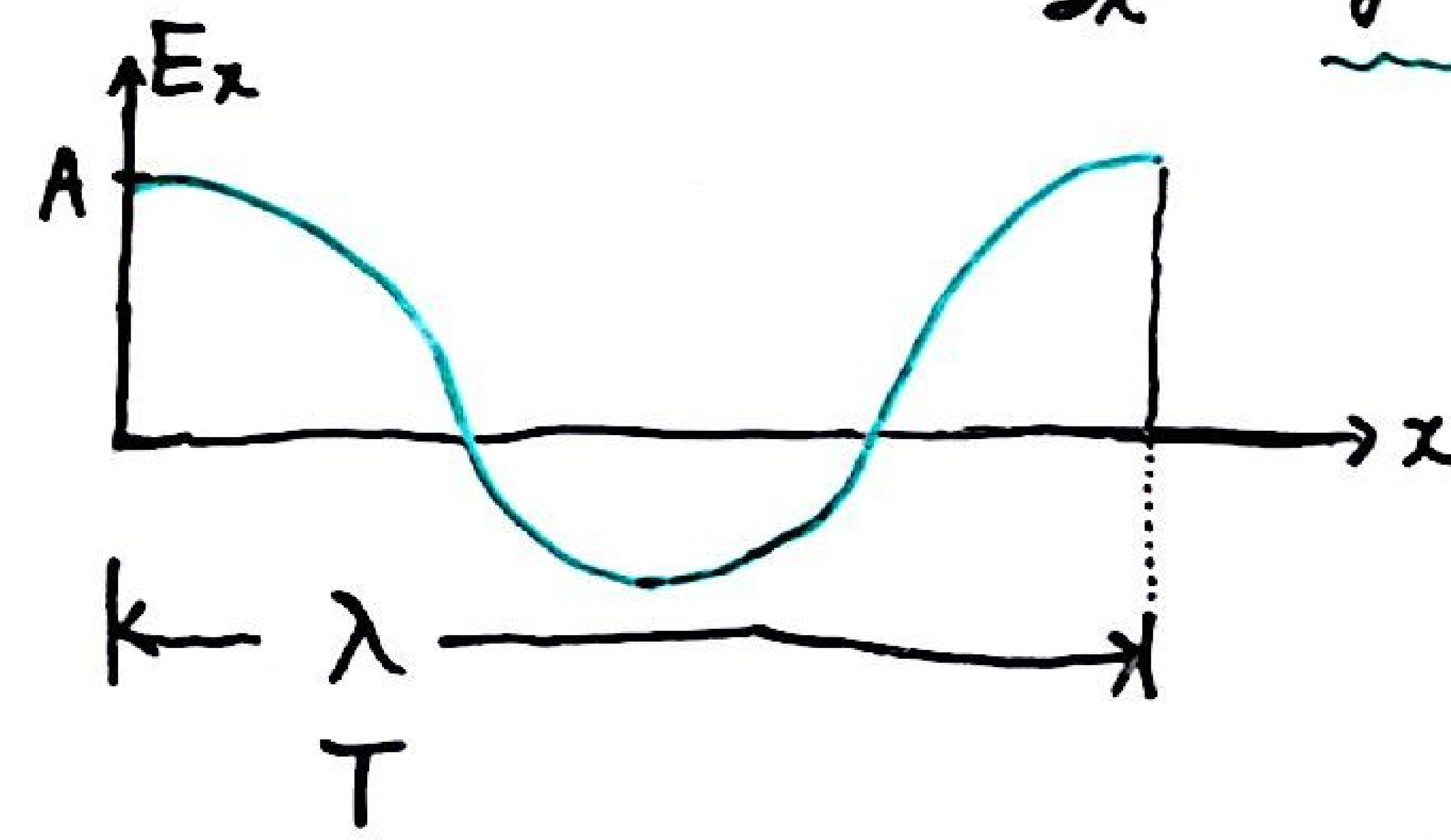
$$= -\mu_0 \frac{\partial}{\partial t}\left(\epsilon_0 \frac{\partial \vec{E}}{\partial t}\right) = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\begin{vmatrix} \nabla & \nabla \\ E & \nabla \\ E & \nabla \end{vmatrix} = (\nabla \cdot \vec{E})\nabla - (\nabla \cdot \nabla)\vec{E} = -\nabla^2 \vec{E}$$

$-$ $+$

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$



$$E_x(x,t) = A\cos(Kx - \omega t)$$

$$\vec{K} = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

wave Vector 각주파수

$$E_x(x,t) = A\cos(Kx-\omega t)$$

$$= A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$

$$= A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

$$\mu_0\epsilon_0 = \frac{K^2}{\omega^2} = \frac{\left(\frac{2\pi}{\lambda}\right)^2}{\left(\frac{2\pi}{T}\right)^2} = \left(\frac{T}{\lambda}\right)^2$$

$$\frac{1}{\mu_0\epsilon_0} = \left(\frac{\lambda}{T}\right)^2$$

$$\frac{\lambda}{T} = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\frac{\partial E_x}{\partial x} = -KA\sin(Kx-\omega t) \qquad \frac{\partial^2 E_x}{\partial x^2} = -K^2\underbrace{A\cos(\quad)}_{E_x} = -K^2 E_x$$

$$= \frac{1}{\sqrt{4\pi\epsilon_0 \; \frac{\mu_0}{4\pi}}}$$

$$\frac{\partial^2 \vec{E}}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\frac{\partial E_x}{\partial t} = +\omega A\sin(Kx-\omega t) \qquad \frac{\partial^2 E_x}{\partial t^2} = -\omega^2\underbrace{A\cos(\quad)}_{E_x} = -\omega^2 E_x$$

$$= \frac{1}{\sqrt{\frac{1}{9\times10^9}\times10^{-7}}}$$

$$-\frac{1}{K^2}\frac{\partial^2 E_x}{\partial x^2} = E_x = -\frac{1}{\omega^2}\frac{\partial^2 E_x}{\partial t^2}$$

$$\mu_0 \to \frac{\mu_0}{4\pi} = 10^{-7}$$

$$= \frac{1}{\sqrt{\frac{1}{9}\times10^{-16}}}$$

$$\frac{\partial^2 E}{\partial x^2} = \frac{K^2}{\omega^2}\frac{\partial^2 E_x}{\partial t^2}$$

$$\epsilon_0 \to 4\pi\epsilon_0 = \frac{1}{9\times10^9}$$

$$= \frac{1}{\frac{1}{3}\times10^{-8}}$$

$$= 3\times10^8 \text{ m/sec}$$

Vacuum ⇒ 에너지가 최소인 상태

$L \Rightarrow \mathcal{L} = T - V(\phi)$

$= \underbrace{\partial^\mu \phi \, \partial_\mu \phi}_{\text{운동에너지 최소}} - \underbrace{\tfrac{1}{2} m^2 |\phi|^2}_{\text{최소}}$

$\partial_\mu \phi = 0$

$\Rightarrow \phi = C$

		Fermion			Boson
quark	$\frac{2}{3}$	u	c	t	γ → 전자기상호작용 $U_Y(1)$
	$-\frac{1}{3}$	d	s	b	W^+, W^-, Z^0 → 약력 $SU_L(2)$
lepton	-1	e	μ	τ	g → 강력 $SU_c(3)$
	0	ν_e	ν_μ	ν_τ	
Higgs					

ϕ

v^2 term

D is J

Definition is Justice

定義 = 正義

$SU_c(3) \times SU_L(2) \times U_Y(1)$

$\hookrightarrow g$ strong, $\hookrightarrow W^\pm, Z^0$ Weak, $\hookrightarrow \gamma$ EM

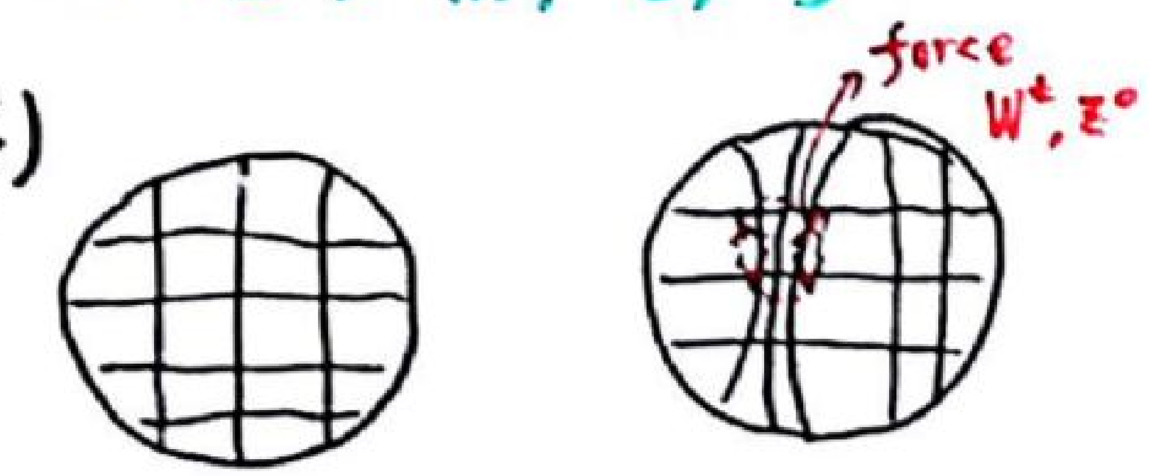
particle ⇒ m, c, s

$|\phi|^2 = \phi^\dagger \phi$

$= \phi^* \phi$

⇒ Global, Local

$V(\phi=0) > V(|\phi| = \frac{v}{\sqrt{2}})$



local gauge symmetry

$V(|\phi|^2) = \mu^2 |\phi|^2 + \lambda |\phi|^4$

$= \underbrace{\mu^2}_{-\lambda v^2} \phi^* \phi + \lambda (\phi^* \phi)^2$

m_Z^2

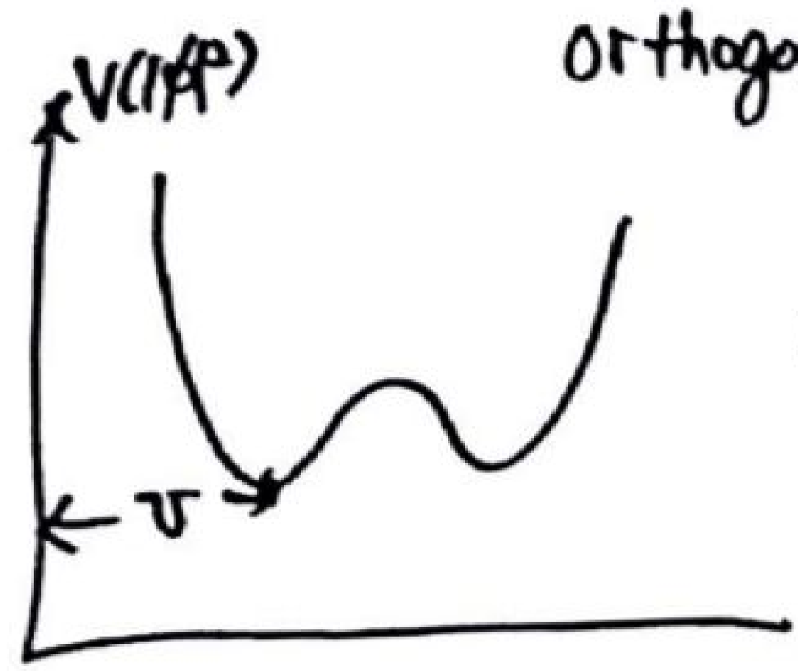
양. → 수 ⇒ Scalar, vector, tensor

orthogonal, Complete

$\frac{\partial V(|\phi|^2)}{\partial \phi^*} = \mu^2 \phi + 2\lambda (\phi^* \phi) \phi = 0$

$m_{W^\pm}^2$

$\mu^2 + 2\lambda \underbrace{\phi^* \phi}_{|\phi|^2} = 0$



$V(\phi=0) = 0$

empty

Void

$\Rightarrow \begin{cases} \phi = 0 \\ |\phi|^2 = \frac{-\mu^2}{2\lambda} = \frac{v^2}{2} \end{cases}$

$v^2 = -\lambda \mu^2$

$\phi = \frac{v}{\sqrt{2}}$

Vacuum expectation value

진공기대값 ⇒ $v = 246$ GeV

$\langle \phi \rangle = \frac{v}{\sqrt{2}}$

$V(|\phi|^2 = \frac{v^2}{2}) = -\lambda v^2 \frac{v^2}{2}$

$+ \lambda (\frac{v^2}{2})^2 = \frac{-\lambda v^4}{2} + \frac{\lambda v^4}{4} = -\frac{\lambda v^4}{4}$

$$\mathcal{L} = (D^\mu\phi)^\dagger D_\mu\phi - V(|\phi|^2) \qquad \mathcal{L} = \frac{1}{2}(D^\mu\underbrace{\phi}_{\to h})^\dagger D_\mu\underbrace{\phi}_{\to h} - \frac{1}{2}\underbrace{m^2}_{\text{질량}}\underbrace{\phi^2}_{\to h^2}$$

$$= \frac{1}{2}\partial^\mu h\,\partial_\mu h + \frac{1}{8}(g_1^2+g_2^2)Z^\mu Z_\mu(h+\underbrace{v}_{v^2})^2 + \frac{1}{4}g_2^2 W^{-\mu}W^+_\mu(h+\underbrace{v}_{v^2})^2 - V(|\phi|^2)$$

$$= \frac{1}{2}\partial^\mu h\,\partial_\mu h + \frac{1}{8}(g_1^2+g_2^2)Z^\mu Z_\mu(h^2+2hv+v^2) + \frac{1}{4}g_2^2 W^{-\mu}W^+_\mu(h^2+2hv+v^2) - V(|\phi|^2)$$

v^2 term $\Rightarrow$ $\frac{1}{8}(g_1^2+g_2^2)v^2 Z^\mu Z_\mu + \frac{1}{4}g_2^2 W^{-\mu}W^+_\mu v^2$

$$= \frac{1}{2}\Big[\underbrace{\frac{1}{4}(g_1^2+g_2^2)v^2}_{m_Z^2}\Big]Z^\mu Z_\mu + \frac{1}{2}v^2\Big[\;g_2^2\big[(W^{+\mu})^* W^+_\mu + W^{-\mu}(W^-_\mu)^*\big]\times\frac{1}{4}$$

$$= \frac{1}{2}m_Z^2 Z^\mu Z_\mu + \frac{1}{2}m_W^2\big[(W^{+\mu})^* W^+_\mu + W^{-\mu}(W^-_\mu)^*\big]$$

$$m_Z^2 = \frac{1}{4}(g_1^2+g_2^2)v^2 \;\rightarrow\; m_Z = \frac{1}{2}v\sqrt{g_1^2+g_2^2}$$

$$m_{W^\pm}^2 = \frac{1}{4}g_2 v^2 \;\rightarrow\; m_{W^\pm} = \frac{1}{2}v g_2$$

$$H = T + V$$

$$L = T - V$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$$

$$\frac{\partial L}{\partial q} = 0 \rightarrow \frac{\partial L}{\partial \dot{q}} = P \rightarrow \text{상수} = \text{보존} = \text{대칭}$$

공간대칭 운동량 보존

$$\frac{\partial L}{\partial t} = 0 \rightarrow \frac{\partial}{\partial t} L(q, \dot{q})$$

시간대칭

$$= \frac{\partial L}{\partial q}\underbrace{\frac{\partial q}{\partial t}}_{\dot{q}} + \frac{\partial L}{\partial \dot{q}}\underbrace{\frac{\partial \dot{q}}{\partial t}}_{\ddot{q}}$$

$$= \dot{q}\frac{\partial L}{\partial q} + \ddot{q}\frac{\partial L}{\partial \dot{q}} = \dot{q}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) + \ddot{q}\frac{\partial L}{\partial \dot{q}}$$

$$\frac{\partial L}{\partial t} = \frac{d}{dt}\left(\dot{q}\frac{\partial L}{\partial \dot{q}}\right) \rightarrow \frac{d}{dt}\left(\dot{q}\underbrace{\frac{\partial L}{\partial \dot{q}}}_{P} - L\right) = 0$$

$$\dot{q}P - L = H \rightarrow \text{상수} = \text{보존} = \text{대칭}$$

$$\frac{d}{dt}(\dot{q}P - L) = 0$$

$$\mathcal{L} = (D^\mu \phi)^\dagger D_\mu \phi - V(|\phi|^2)$$

$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\vec{\tau} = \frac{1}{2}\sigma_i$ $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$g_1 \sin\theta_w = e$

$g_2 \cos\theta_w = e$

$Q = I_3 + \frac{Y}{2}$

Y = hypercharge

$$D_\mu \phi = \partial_\mu + \frac{1}{2} i g_1 Y B_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu$$

$$(B_\mu, W_\mu^i) \to (A_\mu, Z_\mu)$$

$Y = 1$

$u \to \frac{2}{3} = \frac{1}{2} + \frac{Y}{2}$

$Y = \frac{1}{3}$

$$= \partial_\mu + \frac{1}{2} i g_1 Y B_\mu + \frac{1}{2} i g_2 (\sigma_1 W_\mu^1 + \sigma_2 W_\mu^2 + \sigma_3 W_\mu^3)$$

$$W_\mu^3 = Z_\mu \sin\theta_w + A_\mu \cos\theta_w$$

$$= \partial_\mu + \frac{i}{2}(g_1 Y B_\mu + g_2 \sigma_3 W_\mu^3) + \frac{i}{2} g_2 (\sigma_1 W_\mu^1 + \sigma_2 W_\mu^2)$$

$$g_2 W_\mu^3 = g_2 Z_\mu \sin\theta_w + g_2 A_\mu \cos\theta_w$$

$$= g_2 Z_\mu \sin\theta_w + e A_\mu$$

$$= \partial_\mu + \frac{i}{2}\left[\begin{pmatrix} g_1 Y B_\mu & 0 \\ 0 & g_1 Y B_\mu \end{pmatrix} + \begin{pmatrix} g_2 W_\mu^3 & 0 \\ 0 & -g_2 W_\mu^3 \end{pmatrix}\right] + \frac{i}{2} g_2 \left[\begin{pmatrix} 0 & W_\mu^1 \\ W_\mu^1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -iW_\mu^2 \\ iW_\mu^2 & 0 \end{pmatrix}\right]$$

$\sin^2\theta_w \approx 0.23$

$$B_\mu = -Z_\mu \sin\theta_w + A_\mu \cos\theta_w$$

$$= \partial_\mu + \frac{i}{2}\begin{pmatrix} g_1 Y B_\mu + g_2 W_\mu^3 & 0 \\ 0 & g_1 Y B_\mu - g_2 W_\mu^3 \end{pmatrix} + \frac{i}{2} g_2 \begin{pmatrix} 0 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & 0 \end{pmatrix}$$

$$g_1 B_\mu = -g_1 Z_\mu \sin\theta_w + g_1 A_\mu$$

$$+ e A_\mu$$

$$= \partial_\mu + \frac{i}{2}\begin{pmatrix} -g_1 Z_\mu \sin\theta + eYA_\mu + g_2 Z_\mu \cos\theta + eA_\mu & 0 \\ 0 & -g_1 Z_\mu Y \sin\theta + eYA_\mu - g_2 Z_\mu \cos\theta - eA_\mu \end{pmatrix} + \frac{i}{2} g_2 \sqrt{2} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix}$$

$$D_\mu = \partial_\mu + \frac{i}{2}\left[e\begin{pmatrix} Y+1 & 0 \\ 0 & Y-1 \end{pmatrix} A_\mu + \begin{pmatrix} -g_1 Y \sin\theta + g_2 \cos\theta & 0 \\ 0 & -g_1 Y \sin\theta - g_2 \cos\theta \end{pmatrix} Z_\mu\right] + \frac{i g_2}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix}$$

$$= \partial_\mu + ie\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} A_\mu + \begin{pmatrix} -g_1 \sin\theta + g_2 \cos\theta & 0 \\ 0 & -g_1 \sin\theta - g_2 \cos\theta \end{pmatrix} Z_\mu + \frac{i}{\sqrt{2}} g_2 \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix}$$

$g_1 = \sqrt{g_1^2 + g_2^2} \sin\theta_w$ $g_2 = \sqrt{g_1^2 + g_2^2} \cos\theta_w$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$$= \partial_\mu + ie\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} A_\mu + \begin{pmatrix} \sqrt{g_1^2 + g_2^2} \cos 2\theta & 0 \\ 0 & -\sqrt{g_1^2 + g_2^2} \end{pmatrix} Z_\mu + \frac{i}{\sqrt{2}} g_2 \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix}$$

$$\boxed{\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h + v \end{pmatrix}} \leftarrow \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \xi_3 + i\xi_4 \\ \xi_1 + i\xi_2 \end{pmatrix}$$

$\eta = \phi - \langle\phi\rangle$

$\phi = \eta + \langle\phi\rangle$

$(2 \times 2)(2 \times 1) = 2 \times 1$

$$D_\mu \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} + \left[ie\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} A_\mu + \begin{pmatrix} \sqrt{} \cos 2\theta & 0 \\ 0 & -\sqrt{} \end{pmatrix} Z_\mu\right] \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h+v \end{pmatrix}$$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} A_\mu$

$$+ \frac{i}{\sqrt{2}} g_2 \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h+v \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} + \frac{i}{2\sqrt{2}}\begin{pmatrix} 0 \\ -\sqrt{g_1^2 + g_2^2} \end{pmatrix} Z_\mu (h+v) + \frac{i}{2} g_2 \begin{pmatrix} W_\mu^+ \\ 0 \end{pmatrix} (h+v)$$

$$(D^\mu \phi)^\dagger = \frac{1}{\sqrt{2}}(0 \quad \partial^\mu h) + \frac{-i}{2\sqrt{2}}(0 \quad \sqrt{g_1^2 + g_2^2}) Z^\mu (h+v) + \frac{-i}{2} g_2 (W^{-\mu}, 0)(h+v)$$

$$(D^\mu \phi)^\dagger D_\mu \phi = \frac{1}{2}\partial^\mu h \, \partial_\mu h + \frac{1}{8}(g_1^2 + g_2^2) Z^\mu Z_\mu (h+v)^2 + \frac{1}{4} g_2^2 W^{-\mu} W_\mu^+ (h+v)^2$$