$$\delta S = 0$$

$$H = T + V$$

$$L = T - V$$

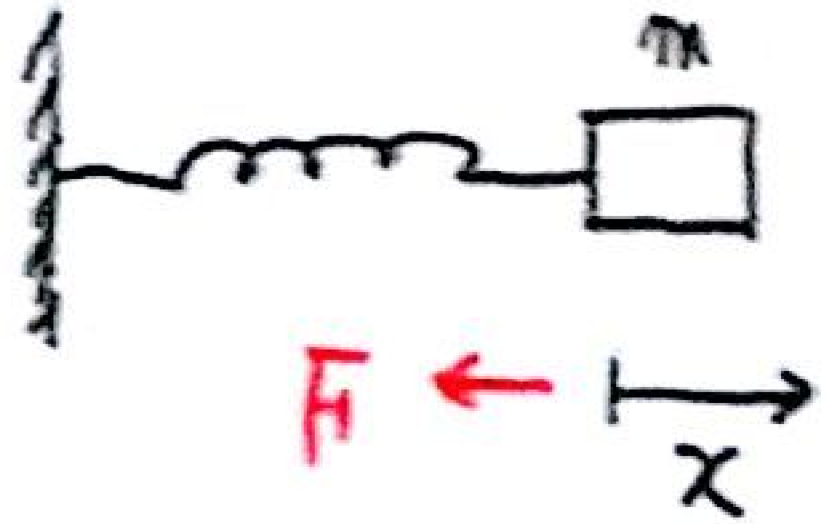
$$H = T + V = \frac{p^2}{2m} + V$$

$$\vec{p} = -i\hbar\vec{\nabla} \qquad p^2 = (-i\hbar\nabla)^2 = -\hbar^2\nabla^2$$

$$H = \frac{-\hbar^2}{2m}\nabla^2 + V$$

$$H\Psi = E\Psi \qquad -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E_n\psi$$

$$L = T - V$$



$$L = \frac{1}{2}m(\dot{x})^2 - \frac{1}{2}kx^2$$

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

$$-kx - \frac{d}{dt}(m\dot{x}) = 0$$

$$m\ddot{x} = -kx$$

$$F = ma$$

$$F = -kx$$

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) = 0$$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\frac{\partial \mathcal{L}}{\partial A_\lambda} = -j^\lambda$$

$$\frac{\partial \mathcal{L}}{\partial A_\lambda} - \partial_\alpha\left(\frac{\partial \mathcal{L}}{\partial(\partial_\alpha A_\lambda)}\right) = 0$$

$$-j^\lambda + \partial_\alpha (\partial^\alpha A^\lambda - \partial^\lambda A^\alpha = 0$$

$$-j^\lambda + \partial_\alpha F^{\alpha\lambda} = 0$$

$$j^\lambda = \partial_\alpha F^{\alpha\lambda}$$

$$j^\nu = \partial_\mu F^{\mu\nu}$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\alpha A_\lambda)} = \frac{\partial}{\partial(\partial_\alpha A_\lambda)}\left[-\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\underline{\partial_\mu} A_\nu - \partial_\nu A_\mu)\right]$$

$$= -\frac{1}{4}\left[\underline{\delta^\alpha_\mu \delta^\lambda_\nu}(\partial^\alpha A^\lambda - \partial^\lambda A^\alpha) - \delta^\alpha_\nu \delta^\lambda_\mu (\partial^\lambda A^\alpha - \partial^\alpha A^\lambda)\right] \times 2$$

$$= -\frac{1}{2} \times 2 (\partial^\alpha A^\lambda - \partial^\lambda A^\alpha)$$

$$\delta S = 0$$

$$S = \int \mathcal{L}(\phi, \partial_\mu \phi, x^\mu)\, d^4x$$

$$\delta S = \delta \int \mathcal{L}(\phi, \partial_\mu \phi, x^\mu)\, d^4x$$

$$= \int \left[\frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \underbrace{\delta(\partial_\mu \phi)}_{\to\ \partial_\mu(\delta\phi)}} \right] d^4x$$

$$\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\delta\phi\right) = \delta\phi\, \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) + \underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\partial_\mu(\delta\phi)}$$

$$= \int \left[\frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\delta\phi\right) - \delta\phi\, \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) \right] d^4x$$

$$= \int \left[\frac{\partial \mathcal{L}}{\partial \phi}\delta\phi - \delta\phi\, \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) \right] d^4x + \int_i^f \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\delta\phi\right) d^4x = 0$$

$$= \int \delta\phi \left[\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) \right] d^4x + \underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\delta\phi \Big|_i^f}_{\delta\phi|_i^f \to 0}$$

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) = 0$$

입자

ψ spinor $\qquad \mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$

$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m_q) q$

$A_\mu \qquad \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$

$B_\mu \qquad \mathcal{L} = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - j^\mu B_\mu + m^2 B^\mu B_\mu$

$\phi \qquad \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2$

$\phi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2) \qquad \mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - \frac{1}{2} m^2 \phi^* \phi$

$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad \mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi - \phi^\dagger M_\phi^2 \phi$

$W_\mu^i \qquad \mathcal{L} = -\frac{1}{4} W^{\mu\nu} W_{\mu\nu} - j^\mu W_\mu + W^\mu M_W^2 W_\mu$

local $U(1)$ gauge transform

$\xrightarrow{} B_\mu$ $\quad \xrightarrow{} e^{i\theta}$

$$\psi \to \psi' = e^{ia\theta(x)}\psi \to \mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

$$D_\mu = \partial_\mu + iag_1 B_\mu$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \to q' = e^{ia_q\theta} q \to \mathcal{L} = \bar{q}(i\gamma^\mu D_\mu - m_q) q$$

$$D_\mu = \partial_\mu + ia_q g_1 B_\mu$$

$$B_\mu \to B'_\mu = B_\mu - \frac{1}{g_1}\partial_\mu \theta \to \mathcal{L} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$

$$\phi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2) \to \phi' = e^{ia_0\theta}\phi \to \mathcal{L} = D^\mu \phi^* D_\mu \phi - m^2 \phi^* \phi$$

$$D_\mu = \partial_\mu + ia_0 g_1 B_\mu$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to \phi' = e^{ia_\phi\theta}\phi \to \mathcal{L} = D^\mu \phi^\dagger D_\mu \phi - \phi^\dagger M_\phi^2 \phi$$

$$D_\mu = \partial_\mu + ia_\phi g_1 B_\mu$$

$\nearrow W_\mu \quad \nearrow B_\mu \quad \nearrow e^{i\alpha}$

local $SU(2)_L \times U(1)_Y$ gauge symmetry $\mathcal{L}$ $\quad q = \frac{Y}{2}$

$\tau_i = \frac{\sigma_i}{2}$

$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $\quad Q = I_3 + \frac{Y}{2}$ $\quad \mathcal{L} = \bar{q}_L (i\gamma^\mu D_\mu - m_q) q_L$

$\frac{2}{3} = \frac{1}{2} + \frac{Y}{2} \to Y = \frac{1}{3}$

$-\frac{1}{3} = -\frac{1}{2} + \frac{Y}{2} \to Y = \frac{1}{3}$ $\quad D_\mu = \partial_\mu + i\frac{1}{6} g_1 B_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu$

$u_R \quad \frac{2}{3} = 0 + \frac{Y}{2} \to Y = \frac{4}{3}$ $\quad \mathcal{L} = \bar{u}_R (i\gamma^\mu D_\mu) u_R$

$D_\mu = \partial_\mu + i\frac{2}{3} g_1 B_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu$

$d_R \quad -\frac{1}{3} = 0 + \frac{Y}{2} \to Y = -\frac{2}{3}$ $\quad \mathcal{L} = \bar{d}_R (i\gamma^\mu D_\mu) d_R$

$(\cdots) \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} \begin{pmatrix} \vdots \end{pmatrix} \leftarrow \bar{\Psi} \gamma^\mu \Psi = \frac{p^\mu}{m} \bar{\Psi} \Psi$ $\quad D_\mu = \partial_\mu - i\frac{1}{3} g_1 B_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu$

$=$ number

$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ $\quad 0 = \frac{1}{2} + \frac{Y}{2} \quad Y = -1$ $\quad \mathcal{L} = \bar{l}_L (i\gamma^\mu D_\mu) l_L$

$-1 = -\frac{1}{2} + \frac{Y}{2} \quad Y = -1$

$D_\mu = \partial_\mu - i\frac{1}{2} g_1 B_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu$

$e_R \quad -1 = 0 + \frac{Y}{2} \quad Y = -2$ $\quad \mathcal{L} = \bar{e}_R (i\gamma^\mu D_\mu) e_R$ $\quad D_\mu = \partial_\mu - i g_1 B_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu$

$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ $\quad 1 = \frac{1}{2} + \frac{Y}{2} \quad Y = 1$ $\quad \mathcal{L} = (D^\mu \phi)^\dagger D_\mu \phi$

$0 = -\frac{1}{2} + \frac{Y}{2} \quad Y = 1$

$D_\mu = \partial_\mu + i\frac{1}{2} g_1 B_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu$

$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\h+v\end{pmatrix}$ $D_\mu \phi$

$\mathcal{L} = (D^\mu\phi)^\dagger D_\mu\phi - \frac{1}{4}\lambda(h^4 + 4h^3v + 4h^2v^2)$

$D_\mu\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \partial_\mu h\end{pmatrix} + \frac{i}{2\sqrt{2}}(g_1^2+g_2^2)^{\frac{1}{2}} Z_\mu (h+v) + \frac{i}{2}g_2 W^{-\mu}(h+v)$

$= \frac{1}{2}\partial^\mu h\,\partial_\mu h - \frac{1}{2}m^2 h^2 + \cdots$

$(D^\mu\phi)^\dagger = \frac{1}{\sqrt{2}}(0 \;\; \partial^\mu h) + \frac{-i}{2\sqrt{2}}(g_1^2+g_2^2)^{\frac{1}{2}} Z^\mu(h+v) + \frac{-i}{2}g_2 W_\mu^+(h+v)$

$= \frac{1}{2}\partial^\mu h\,\partial_\mu h - \lambda v^2 h^2$

$\hookrightarrow \frac{1}{2}(2\lambda v^2)h^2$

$\hookrightarrow m_h^2$

$(D^\mu\phi)^\dagger D_\mu\phi = \frac{1}{2}\partial^\mu h\,\partial_\mu h + \frac{1}{8}(g_1^2+g_2^2)Z^\mu Z_\mu (h+v)^2 + \frac{1}{4}g_2^2 W^{-\mu}W_\mu^+(h+v)^2$

$\mathcal{L} = (D^\mu\phi)^\dagger D_\mu\phi - V(|\phi|^2)$ $\quad |\phi|^2 = \phi^*\phi$ $\quad |\phi| = \frac{v}{\sqrt{2}}$ 진공기대치 $\quad m_h = \sqrt{2\lambda v^2}$

$V(|\phi|^2) = \mu^2|\phi|^2 + \lambda|\phi|^4$

$\begin{cases}\phi = 0\\ |\phi|^2 = \frac{-\mu^2}{2\lambda} = \frac{v^2}{2}\end{cases}$

$V(\phi=0) > V(|\phi| = \frac{v}{\sqrt{2}})$

$\mu^2 = -\lambda v^2$

$\frac{\partial V}{\partial\phi^*} = \frac{\partial[\mu^2\phi^*\phi + \lambda(\phi^*\phi)^2]}{\partial\phi^*} = \mu^2\phi + 2\lambda|\phi|^2\phi = 0$

$|\phi|^2 = \phi^\dagger\phi = \frac{1}{\sqrt{2}}(0 \;\; h+v)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\h+v\end{pmatrix}$

$= \frac{1}{2}(h+v)^2$

$V(|\phi|^2) - \frac{v^4}{4} = -\lambda v^2|\phi|^2 + \lambda|\phi|^4$ $\quad V(|\phi|^2) = \lambda\left(|\phi|^4 - v^2|\phi|^2 + \frac{v^4}{4}\right)$

$= \lambda\left(|\phi|^2 - \frac{v^2}{2}\right)^2$

$V(|\phi|^2) = \lambda\left[\frac{1}{2}(h+v)^2 - \frac{v^2}{2}\right]^2 = \lambda\left[\frac{1}{2}(h^2+2hv+v^2) - \frac{v^2}{2}\right]^2 = \lambda\left[\frac{1}{2}(h^2+2hv)\right]^2$

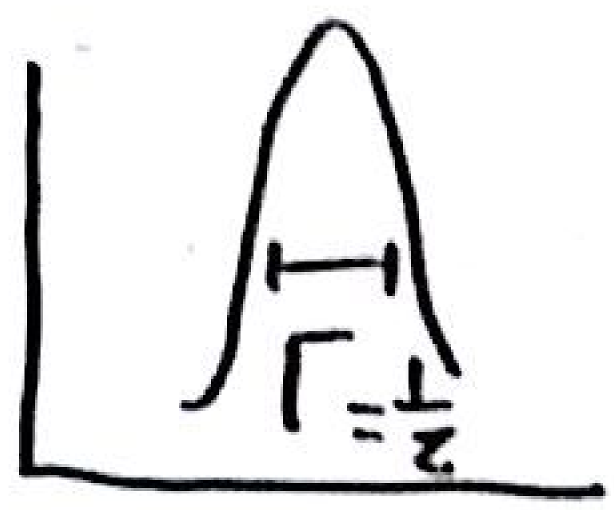
$= \frac{1}{4}\lambda(h^4 + 4h^3v + 4h^2v^2)$

$m_{W^\pm} = \frac{1}{2} v g_2$ $\quad m_h = \sqrt{2\lambda v^2}$

$m_Z = \frac{1}{2} v \sqrt{g_1^2 + g_2^2}$

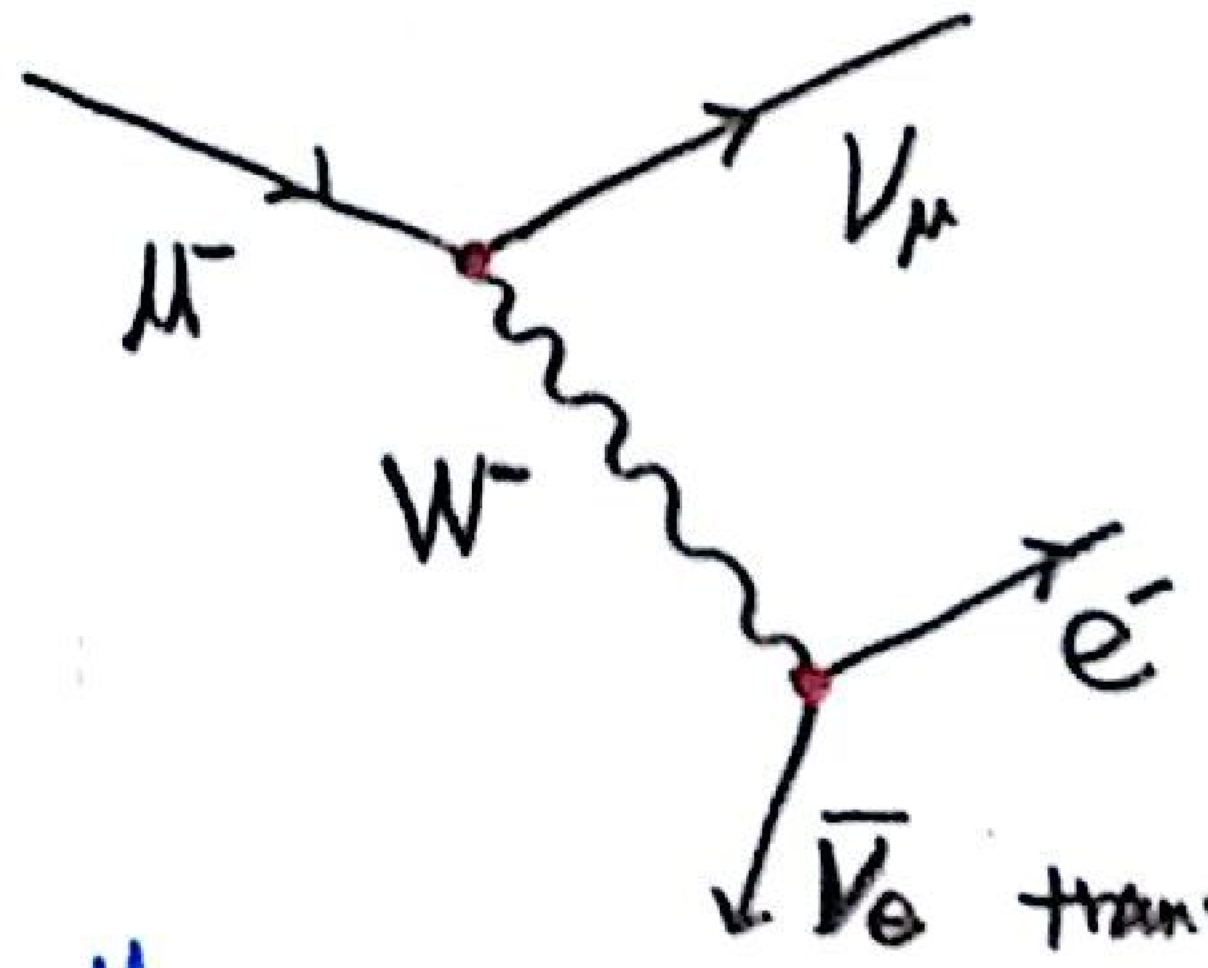
$$g_2 = \frac{2M_W}{v}$$

Vertices $\rightarrow g_2 \bar{u} \gamma^\mu P_L u$



$$\mu^- \longrightarrow \nu_\mu e^- \bar{\nu}_e$$

propagator of W^- $\quad \frac{1}{Q - M_W^2} \quad P_L = \frac{1-\gamma^5}{2}$



transfer matrix
전이행렬

$$M = \left(\frac{g_2}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu P_L \mu \right) \frac{1}{Q - M_W^2} \left(\frac{g_2}{\sqrt{2}} \bar{e} \gamma^\mu P_L \nu_e \right)$$

$$= \frac{g_2^2}{8M_W^2} \bar{\nu}_\mu \gamma^\mu (1-\gamma^5) \mu \, \bar{e} \gamma^\mu (1-\gamma^5) \nu_e$$

$$= \frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1-\gamma^5) \mu \, \bar{e} \gamma^\mu (1-\gamma^5) \nu_e$$

$$\gamma^\mu = \begin{pmatrix} 0 & -\sigma_i \\ \sigma_i & 0 \end{pmatrix}$$

$$\frac{g_2^2}{8M_W^2} = \frac{G_F}{\sqrt{2}} \rightarrow \frac{\left(\frac{2M_W}{v}\right)^2}{8M_W^2} = \frac{1}{2v^2} = \frac{G_F}{\sqrt{2}}$$

$$v = \frac{1}{2^{\frac{1}{4}}\sqrt{G_F}} = \frac{1}{2^{\frac{1}{4}}\sqrt{1.17 \times 10^{-5} \frac{1}{(\mathrm{GeV})^2}}}$$

$$= 246 \ \mathrm{GeV}$$

$$\frac{1}{\tau_\mu} = \Gamma = \frac{G_F^2\, m_\mu^5}{192\pi^3}$$

$\tau = 2.2 \times 10^{-6}$ sec

$$G_F = 1.17 \times 10^{-5} \frac{1}{(\mathrm{GeV})^2}$$

$m_\mu = 105.6$ MeV

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

$m_e = 0.51$ MeV

$m_\tau = 1784$ MeV

$$\mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m_q)q$$