$H_0 = 100h$ km/s·Mpc

$= 3.24 \times 10^{-18} h/\text{sec}$

$h = 0.71$

$1\ ^\circ K = 1.24 \times 10^{-4}\ eV$

$300^\circ K = 10\ \mu m$

$\lambda_{max} T = 0.289\ cm \cdot {}^\circ K$

$\Omega_0 = 1.02$

$1\ Au = 1.5 \times 10^{11}\ m$

$1\ ly = 9.5 \times 10^{15}\ m$

$1\ Pc = 3.1 \times 10^{16}\ m$ $T_{dec} = 3000K$

t_{dec} = 37.8만년 $\Omega_{m0} = 0.27$

$Z_{dec} = 1089$ $\Omega_{\Lambda 0} = 0.73$

$\Omega_{b0} = 0.04$

$Z_{rm} = 3233$, T_{rm} = 5만 5천년

$T_0 = 2.725\ ^\circ K$

$300^\circ K = 26\ meV$

$1^\circ K$, 1ℓ = 2만개

$3\ ^\circ K$. $1\ell = 2 \times 10^4 \times 3^3 = 5.5 \times 10^5$ photons

$$P_i = \Omega_i \quad \Omega_i = e^{\frac{S_i}{K}}$$

$$S_i = K \ln \Omega_i$$

$$\Omega_0 = \Omega_R \Omega_\Delta \quad \Omega_R = e^{\frac{1}{K}[\Delta E - \beta \Delta n]}$$

$$P_i \to \Omega_i \quad \Omega_i \to \bar{n} \to dN \to dE \to du$$

$$u = aT^4$$

$$\frac{du}{d\varepsilon} = \frac{8\pi}{c^3 h^3} \frac{\varepsilon^3}{e^{\beta\varepsilon} - 1}$$

$$\frac{du}{d\lambda} = 8\pi ch \frac{\lambda^{-5}}{e^{\beta hc/\lambda} - 1}$$

$$\frac{du}{d\omega} = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\beta\hbar\omega} - 1}$$

$$\frac{du}{d\lambda} = 0 \quad \lambda_{max} T = 0.289\ cm \cdot {}^\circ K$$

빈의 변위공식

$$(ds)^2 = (cdt)^2 - R^2(t)\left[\left(\frac{dw}{\sqrt{1-Kw^2}}\right)^2 + (w\,d\theta)^2 + (w \sin\theta\, d\phi)^2\right]$$

FRW metric

$r(t) = R(t)\,w$

$(ds)^2 = 0$

null interval

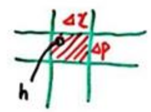
$h = 6.626 \times 10^{-34}$ J·sec

$= 4.14 \times 10^{-15}$ eV·sec

$E = h\nu$

$\omega = 2\pi\nu$

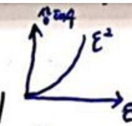
$\hbar = \frac{h}{2\pi}$



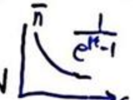
$K = 6.8 \times 10^{-5}$ eV/°K

LSS

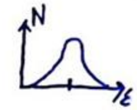
$$U = Q - W + \mu N$$



$$du = dQ - dw + \mu dN$$



$$du = Tds - pdv + \mu dN$$

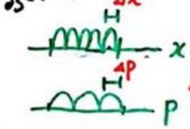


$$U(s + \Delta Q) = U(s) + \frac{\partial u}{\partial s}(\Delta Q) + \frac{\partial^2 u}{\partial s^2}(\Delta Q)^2 + \cdots$$

$$du = \frac{\partial u}{\partial s}(\Delta Q)$$

$= \frac{1}{T}$

ideal gas



$\Delta x \Delta p \geq h$

$$\Omega_1 = \frac{\Delta x_1 \Delta y_1 \Delta z_1\ \Delta P_{1x} \Delta P_{1y} \Delta P_{1z}}{h^3}$$

$$\Omega_N = \Omega_1 \Omega_2 \cdots \Omega_N = \prod_i \Omega_i$$

$$\Omega_N = \prod_i \Omega_i = \frac{1}{h^{3N}} \prod_i \iint dx_i\, dy_i\, dz_i\, dp_{ix}\, dp_{iy}\, dp_{iz}$$

$$= \frac{V^N}{h^{3N}} \prod_i \int dp_{ix}\, dp_{iy}\, dp_{iz} = \frac{V^N}{h^{3N}} \frac{\pi^{\frac{3N}{2}}}{\frac{3N}{2}\left(\frac{3N}{2} - 1\right)!} (2m$$

$\to C$

$$= C\, V^N (2mu)^{\frac{3N}{2}}$$

$\Omega_N = C V^N (2mu$

$S = K \ln \Omega_N = K \ln [$

$= K \ln C + KN \ln$

$= C + KN \ln V +$

U. N 일정

$du = Tds - pdv$

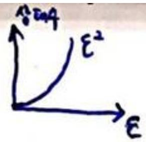
$Tds = pdv$

$\frac{E_i - E_f}{h} = \nu$ $E = h\nu$

$c = \lambda\nu$

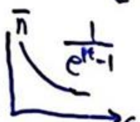
$\ldots)^2 + (w\,d\theta)^2 + (w \sin\theta\, d\phi)^2]$

$U = Q - W + \mu N$



$\Omega_N = C V^N (2mU)^{\frac{3N}{2}}$

$du = dQ - dW + \mu dN$



$S = K \ln \Omega_N = K \ln [C V^N (2mU)^{\frac{3N}{2}}]$

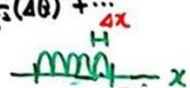
$du = Tds - pdv + \mu dN$

$= K \ln C + KN \ln V + \frac{3}{2} NK \ln (2mU)$

$U(S+\Delta Q) = U(S) + \frac{\partial U}{\partial S}(\Delta Q) + \frac{\partial^2 U}{\partial S^2}(\Delta Q)^2 + \cdots$

$du = \frac{\partial U}{\partial S}(\Delta Q)$

$\frac{\partial U}{\partial S} = \frac{1}{T}$



$= C + KN \ln V + \frac{3}{2} NK \ln U$

$\Delta x \Delta p \geq h$

ideal gas

U. N 일정

sec

.sec

$\Omega_1 = \frac{\Delta x_1 \Delta y_1 \Delta z_1\, \Delta P_{1x} \Delta P_{1y} \Delta P_{1z}}{h^3}$

$du = Tds - pdv + \mu dN$

$Tds = pdv \qquad \frac{ds}{dV} = \frac{P}{T} = NK \frac{1}{V}$

$\Omega_N = \Omega_1 \Omega_2 \cdots \Omega_N = \prod_i \Omega_i$

$\frac{E_i - E_f}{h} = \nu \qquad E = h\nu \qquad PV = NKT$

$C = \lambda \nu$

$\frac{ds}{du} = \frac{3}{2} NK \cdot \frac{1}{U} = \frac{1}{T}$

$\Omega_N = \prod_i \Omega_i = \frac{1}{h^{3N}} \prod_i \iint dx_i dy_i dz_i\, dP_{ix} dP_{iy} dP_{iz}$

$U = \frac{3}{2} NKT$

$\ldots^5$ eV/°K

$= \frac{V^N}{h^{3N}} \prod_i \int dP_{ix} dP_{iy} dP_{iz} = \frac{V^N}{h^{3N}} \frac{\pi^{\frac{3N}{2}}}{\frac{3N}{2}(\frac{3N}{2}-1)!} (2mU)^{\frac{3N}{2}}$

$\to C$

$\frac{U}{N} = \frac{3}{2} KT = \frac{1}{2} m v^2$

$KT = \frac{1}{3} m v^2$

$= C V^N (2mU)^{\frac{3N}{2}}$

$T = 10^9 K. \quad \rho_b = 10^{-2} kg/m^3 \qquad \rho R^4 = \rho_0 \qquad RT = T_0$

$\rho R^{3(1+W)} = \rho_0 \qquad \rho_b R^3 = \rho_0 \qquad \frac{aT^4}{c^2} R^4 = \frac{aT_0^4}{c^4} \qquad du = T\ldots$

$t = 178$

$R = \left(\frac{\rho_0}{\rho_{b0}}\right)^{\frac{1}{3}} = \left(\frac{\times 10^{-24}}{10^{-2} kg}\right)^{\frac{1}{3}} = 3.47 \times 10^{-9} \qquad \frac{du}{dt} = \ldots$

$V\ldots$

$P = \frac{\rho}{m} KT = \frac{\rho c^2}{mc^2} KT = \frac{\rho c^2}{mc^2} \cdot \frac{1}{3} m v^2$

$P = \frac{1}{3} \frac{v^2}{c^2} \rho c^2 \qquad P = w \rho c^2$

$W = \frac{v^2}{3c^2}$

RDU $W = \frac{1}{3}$

MDU $W = 0$

ΛDU $W = -1$

$\dot{u} + \ldots$

$V(\ldots$

$v = 500\ m/sec$

$W = \frac{500^2}{3(3 \times 10^8 m/sec)^2} \cong 10^{-12} \simeq 0$

$H_0 = 3.24 \times 10^{-18} h/sec$

$\frac{1}{H_0} = \frac{1}{3.24 \times 10^{-18} h \cdot sec}$

$= 9.78 \times 10^9 h^{-1} yr = \frac{9.78 \times 1\ldots}{0.71}$

$= t_H$

T_0 $\quad 3.47\times10^{-9}\times10^{9} = T_0 = 3.47\,K$

$$U = T\,dS - P\,dV + \mu\,dN$$

$\rightarrow 0 \qquad \rightarrow 0$

$$\sinh x = \frac{e^{x}-e^{-x}}{2}$$

$$\frac{dU}{dt} = -P\frac{dV}{dt} \qquad \dot{U} + P\dot{V} = 0$$

$$V = \frac{4\pi}{3}r^3 \qquad \dot{V} = \frac{4\pi}{3}\cdot 3r^2\dot{r} = \frac{\frac{4\pi}{3}r^3}{r}3\dot{r} = \frac{V}{r}3\dot{r}$$

$$\dot{V} = 3\frac{\dot{r}}{r}V \rightarrow r(t) = R(t)\,\omega$$

$$\dot{V} = 3\frac{\dot{R}}{R}V \qquad U = \varepsilon V \qquad \dot{U} = \dot{\varepsilon}V + \varepsilon\dot{V}$$

$$\dot{U} + P\dot{V} = \dot{\varepsilon}V + \varepsilon 3\frac{\dot{R}}{R}V + P3\frac{\dot{R}}{R}V = 0$$

$$V\left(\dot{\varepsilon} + 3\varepsilon\frac{\dot{R}}{R} + 3P\frac{\dot{R}}{R}\right) = 0$$

$$\dot{\varepsilon} + 3\frac{\dot{R}}{R}(\varepsilon + P) = 0 \qquad \varepsilon = \rho c^2 \qquad \dot{\varepsilon} = 0 \qquad \varepsilon = -P$$

$$P = -\varepsilon = -\rho c^2 = -1\,\rho c^2$$

$w = -1$

$\cong 10^{-12} \cong 0$

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{R}}{R} \qquad \ln\rho = -3(1+w)\ln R$$

h/sec

h-sec

$$\rho = \rho_0 R^{-3(1+w)} \qquad R^{3(1+w)}\rho = \rho_0$$

$${}^{9}h^{-1}\,yr = \frac{9.78\times10^{9}}{0.71} = 137\text{억년}$$

$$R^3\rho_m = \rho_0 \qquad \rho_m = \frac{\rho_{m0}}{R^3} \qquad R^4\rho_{rel} = \rho_{rel0} \qquad \rho_{rel} = \frac{\rho_{rel0}}{R^4}$$

$$U = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$U = \frac{1}{2}m(\dot{r})^2 - \frac{Gm}{r}\frac{4\pi}{3}r^3\rho$$

$r(t) = R(t)\,\omega$ ← comving

Scale factor

$0\sim1 \quad R(t_0) = 1$

$$2U = m\dot{R}^2\omega^2 - \frac{8\pi G}{3}R^2\omega^2 m\rho$$

$$\frac{2U}{R^2 m\omega^2} = \left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G}{3}\rho$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{KC^2}{R^2}$$

$$K = \frac{-2U}{mc^2\omega^2}$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{KC^2}{R^2} \qquad \left[\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G}{3}\rho\right]R^2 = -KC^2$$

$$\rho = \rho_0 R^{-3(1+w)}$$

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho_0 R^{-3(1+w)} - \frac{KC^2}{R^2}$$

$R(t) =$

$$R(t) = \left(\frac{t}{t_0}\right)^{x} \qquad K = 0 \qquad v = Hr$$

$$\frac{1}{2}mH^2r^2 = \frac{Gm}{r}\frac{4\pi}{3}r^3\rho_c$$

$$\frac{\dot{R}}{R} = \frac{x\left(\frac{t}{t_0}\right)^{x-1}\frac{1}{t_0}}{\left(\frac{t}{t_0}\right)^{x}} = \frac{\frac{x}{t_0}}{\frac{t}{t_0}} = \frac{x}{t}$$

$$H_0^2 = \frac{8\pi G}{3}\rho_c \qquad \frac{8\pi G}{3} = \frac{H^2}{\rho_c} =$$

$$\frac{\dot{R}}{R} = H(t) = \frac{x}{t} \qquad H_0 = \frac{x}{t_0} \rightarrow t_0 = \frac{1}{H_0}x = t_H x$$

$$\left[\left(\frac{\dot{R}}{R}\right)^2 - \frac{H_0^2}{\rho_{c0}}(\rho_m + \right.$$

$$(\dot{R})^2 \Rightarrow \left[x\left(\frac{t}{t_0}\right)^{x-1}\frac{1}{t_0}\right]^2 = \left[\left(\frac{t}{t_0}\right)^{x}\right]^{-3(1+w)+2}$$

$$\left[\left(\frac{\dot{R}}{R}\right)^3 - \frac{H_0^2}{\rho_{c0}}\left(\frac{\rho_{m0}}{R^3} + \right.\right.$$

$$2(x-1) = [-3(1+w)+2]\,x$$

$$x = \frac{2}{3(1+w)}$$

$$\left[\left(\frac{\dot{R}}{R}\right)^2 - H_0^2\left(\frac{\Omega_{m0}}{R} + \right.\right.$$

RDU $\quad w = \frac{1}{3} \quad x = \frac{1}{2} \quad R(t) = \left(\frac{t}{t_0}\right)^{\frac{1}{2}} \quad t_0 = \frac{1}{2}t_H \qquad \Omega_{m0} = \frac{\rho_{m0}}{\rho_{c0}}$

MDU $\quad w = 0 \quad x = \frac{2}{3} \quad R(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3}} \quad t_0 = \frac{2}{3}t_H \qquad P = \frac{1}{3}aT^4 = \frac{1}{3}\rho c^2$

$$U=\frac{1}{2}mv^2-\frac{GMm}{r}$$

$$U=\frac{1}{2}m(\dot r)^2-\frac{Gm}{r}\frac{4\pi}{3}r^3\rho$$

$r(t)=R(t)\,\omega$ ← comoving

↖ Scale factor

0~1 $R(t_0)=1$

$$2U=m\dot R^2\omega^2-\frac{8\pi G}{3}R^2\omega^2 m\rho$$

$$\frac{2U}{R^2m\omega^2}=\left(\frac{\dot R}{R}\right)^2-\frac{8\pi G}{3}\rho$$

$$\left(\frac{\dot R}{R}\right)^2=\frac{8\pi G}{3}\rho-\frac{KC^2}{R^2}$$

$$K=\frac{-2U}{mc^2\omega^2}$$

$\varepsilon=-P$

$P=-\varepsilon=-\rho c^2=-1\,\rho c^2$

$w=-1$

$\ln R$

$(1+W)$

$$R^{3(1+W)}\rho=\rho_0$$

$$R^3\rho_m=\rho_0\qquad \rho_m=\frac{\rho_{m0}}{R^3}\qquad R^4\rho_{rel}=\rho_{rel0}\qquad \rho_{rel}=\frac{\rho_{rel0}}{R^4}$$

$$\left(\frac{\dot R}{R}\right)^2=\frac{8\pi G}{3}\rho-\frac{KC^2}{R^2}$$

$$\rho=\rho_0R^{-3(1+W)}$$

$$\left(\frac{\dot R}{R}\right)^2=\frac{8\pi G}{3}\rho_0R^{-3(1+W)}-\frac{KC^2}{R^2}$$

$$R(t)=\left(\frac{t}{t_0}\right)^x\qquad K=0\qquad v=Hr$$

$$\frac{\dot R}{R}=\frac{x\left(\frac{t}{t_0}\right)^{x-1}\frac{1}{t_0}}{\left(\frac{t}{t_0}\right)^x}=\frac{\frac{x}{t_0}}{\frac{t}{t_0}}=\frac{x}{t}$$

$$\frac{\dot R}{R}=H(t)=\frac{x}{t}\qquad H_0=\frac{x}{t_0}\rightarrow t_0=\frac{1}{H_0}x=t_Hx$$

$$(\dot R)^2\Rightarrow\left[x\left(\frac{t}{t_0}\right)^{x-1}\frac{1}{t_0}\right]^2=\left[\left(\frac{t}{t_0}\right)^x\right]^{-3(1+W)+2}$$

$$2(x-1)=[-3(1+W)+2]\,x$$

$$x=\frac{2}{3(1+W)}$$

RDU $W=\frac{1}{3}$ $x=\frac{1}{2}$ $R(t)=\left(\frac{t}{t_0}\right)^{\frac{1}{2}}$ $t_0=\frac{1}{2}t_H$

MDU $W=0$ $x=\frac{2}{3}$ $R(t)=\left(\frac{t}{t_0}\right)^{\frac{2}{3}}$ $t_0=\frac{2}{3}t_H$

$$\left[\left(\frac{\dot R}{R}\right)^2-\frac{8\pi G}{3}\rho\right]R^2=-KC^2$$

$$t=\frac{2}{3H_0\sqrt{\Omega_{\Lambda0}}}\ln\left[\sqrt{\left(\frac{\Omega_{\Lambda0}}{\Omega_{m0}}\right)R^3+1}+\sqrt{\left(\frac{\Omega_{\Lambda0}}{\Omega_{m0}}\right)R^3}\right.$$

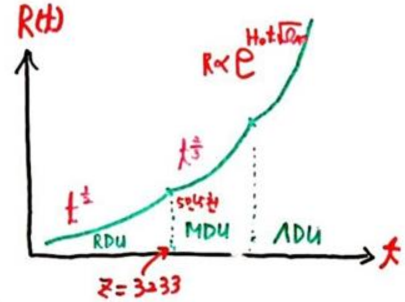
m, M

$$\frac{1}{2}mv^2-\frac{GMm}{r}=0$$

$$R(t)=\left(\frac{\Omega_{m0}}{\Omega_{\Lambda0}}\right)^{\frac{1}{3}}\sinh^{\frac{2}{3}}\left(\frac{3}{2}H_0t\sqrt{\Omega_{\Lambda0}}\right)$$

$$\frac{1}{2}mH^2r^2=\frac{Gm}{r}\frac{4\pi}{3}r^3\rho_c$$

$$H_0^2=\frac{8\pi G}{3}\rho_c\qquad \frac{8\pi G}{3}=\frac{H^2}{\rho_c}=\frac{H_0^2}{\rho_{c0}}$$



$$\left[\left(\frac{\dot R}{R}\right)^2-\frac{H_0^2}{\rho_{c0}}(\rho_m+\rho_{rel}+\rho_\Lambda)\right]R^2=0$$

$$R(t)=\left(\frac{\Omega_{m0}}{4\Omega_{\Lambda0}}\right)\left(e^{\frac{3}{2}H_0t\sqrt{\Omega_{\Lambda0}}}-e^{-\frac{3}{2}H_0t\sqrt{\Omega_{\Lambda0}}}\right)$$

$$\left[\left(\frac{\dot R}{R}\right)^3-\frac{H_0^2}{\rho_{c0}}\left(\frac{\rho_{m0}}{R^3}+\frac{\rho_{rel0}}{R^4}+\rho_{\Lambda0}\right)\right]R^2=0$$

$$R(t)\propto e^{\frac{3}{2}H_0t\sqrt{\Omega_{\Lambda0}}}$$

$$\left[\left(\frac{\dot R}{R}\right)^2-H_0^2\left(\frac{\Omega_{m0}}{R}+\underbrace{\frac{\Omega_{rel0}}{R^2}}_{\approx10^{-4}\approx0}+\Omega_{\Lambda0}\right)\right]=0$$

$$\Omega_{m0}=\frac{\rho_{m0}}{\rho_{c0}}\qquad \Omega_{rel}=\frac{\rho_{rel0}}{\rho_{c0}}\qquad \Omega_{\Lambda0}=\frac{\rho_{\Lambda0}}{\rho_{c0}}$$

$$P=\frac{1}{3}aT^4=\frac{1}{3}\rho c^2\qquad \rho=\frac{aT^4}{c^2}$$