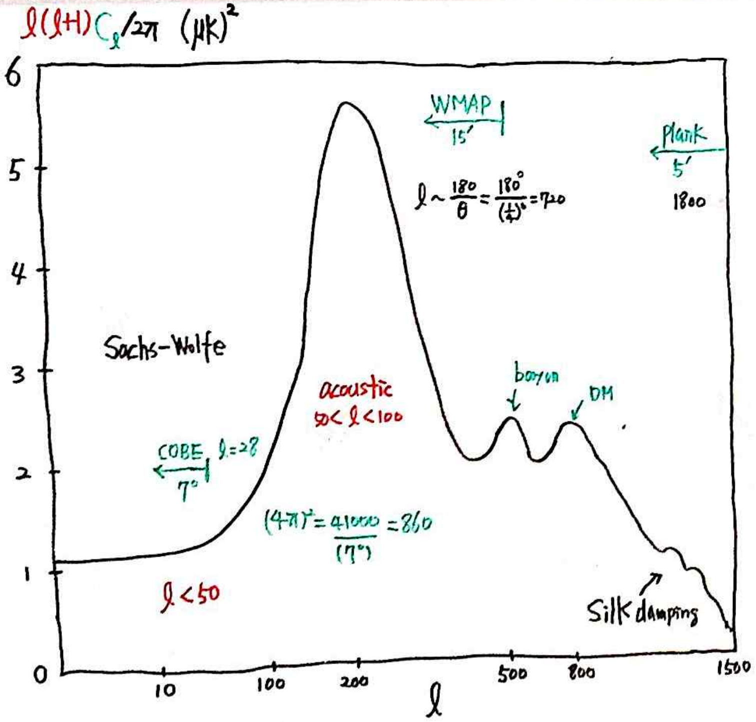


$$\langle T \rangle = 2.725^\circ K$$

$$\left\langle \left(\frac{\delta T}{T}\right)^2 \right\rangle^{\frac{1}{2}} = 1.1 \times 10^{-5}$$

$$\frac{\delta T(\theta,\phi)}{T} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}$$

$$Y_{\ell m} = \sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\, P_\ell(\cos\theta)\, e^{im\phi}$$

$$\langle a_{\ell m} \rangle = 0 \qquad \langle a^*_{\ell m} a_{\ell' m'} \rangle = C_\ell \delta_{\ell\ell'} \delta_{mm'}$$

$$C(\theta) = \frac{1}{4\pi} \sum_{\ell=0}^{\infty} (2\ell+1) C_\ell P_\ell(\cos\theta)$$

$$C_\ell = \frac{2}{\pi} \int_0^\infty dk\, k^2 P(k) \frac{|\Theta_\ell(k)|^2}{|\delta(k)|^2}$$

monopole

$$\Theta_\ell(\eta_0) = [\Theta_0(\eta_*) + \Psi(\eta_*)]\, j_\ell(k(\eta_0-\eta_*))$$

dipole

$$+ 3\Theta_1(\eta_*)\left[j_{\ell-1}(k(\eta_0-\eta_*)) - \frac{\ell+1}{k(\eta_0-\eta_*)} j_\ell(k(\eta_0-\eta_*)) \right]$$

$$+ \int_0^{\eta_0} e^{-\tau} [\dot\Psi(\eta) - \dot\Phi(\eta)]\, j_\ell(k(\eta_0-\eta))\, d\eta$$

SW effect

$$\Theta_0(\eta_*) + \Psi(\eta_*) = -\frac{1}{3}\Phi(\eta_*)$$

$$P(k) = 2\pi^2 \delta_H^2 \frac{k^n}{H_0^{n+3}} T^2(k) \left(\frac{D(\eta)}{D(\eta_0)}\right)^2$$

$$\Phi(\eta_*) = \frac{3}{2k^2} H_0^2 \Omega_{m0} D^{-1}(a_0)\, \delta(\eta)$$

$$\ell(\ell+1) C_\ell^{LSW}(\eta_0) = \frac{\pi \Omega_{m0}}{2 D^2(a_0)} \delta_H^2$$

$$C_\ell(\eta_0) = \frac{2}{\pi} \int_0^\infty dk\, k^2\, 2\pi^2 \delta_H^2 \frac{k^n}{H_0^{n+3}} \left[\frac{1}{2k^2} H_0^2 \Omega_{m0} D^{-1}(a_0)\, \delta(\eta_0)\right]^2 \times j_\ell^2(k(\eta_0-\eta_*)) \frac{1}{\delta(\eta_0)^2}$$

$$C_\ell(\eta_0) = \frac{\pi \Omega_{m0}}{D^2(a_0) H_0^{n-1}} \delta_H^2 \int_0^\infty dk\, k^{n-2} j_\ell^2(k(\eta_0-\eta_*))$$

$$k(\eta_0-\eta_*) \equiv x \quad dk = \frac{dx}{\eta_0} \quad k = \frac{x}{\eta_0}$$

$$\eta_0 \gg \eta_*$$

$$C_\ell(\eta_0) = \frac{\pi \Omega_{m0}}{D^2(a_0) H_0^{n-1}} \delta_H^2 \int_0^\infty \left(\frac{dx}{\eta_0}\right) \left(\frac{x}{\eta_0}\right)^{n-2} j_\ell^2(x).$$

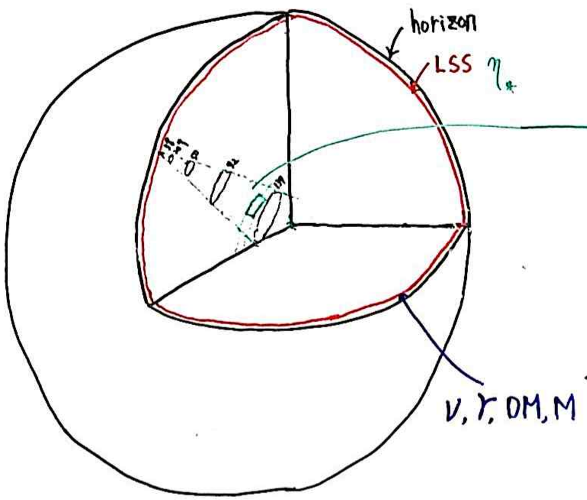
$$= \frac{\pi \Omega_{m0}}{D^2(a_0) (\eta_0 H_0)^{n-1}} \delta_H^2 \int_0^\infty dx\, x^{n-2} j_\ell^2(x)$$

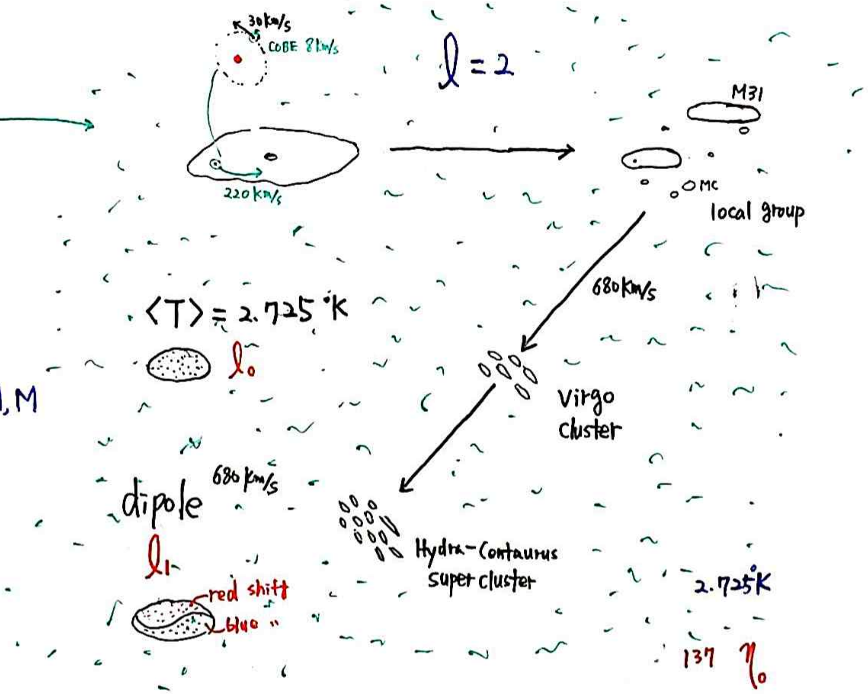
$$j_\ell(x) = \sqrt{\frac{\pi/2}{x}}\, J_{\ell+\frac{1}{2}}(x)$$

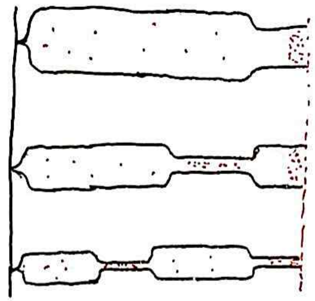
$$= \frac{\pi^2 \Omega_{m0}}{2 D^2(a_0) (\eta_0 H_0)^{n-1}} \delta_H^2 \int_0^\infty \underbrace{dx\, x^{n-3} J^2_{\ell+\frac{1}{2}}(x)}_{\to \frac{1}{\pi \ell(\ell+1)}}$$

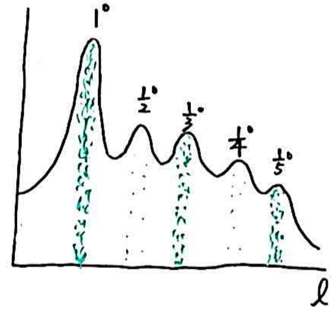
$n = 1$ Harrison, Zeldovich, Peebles

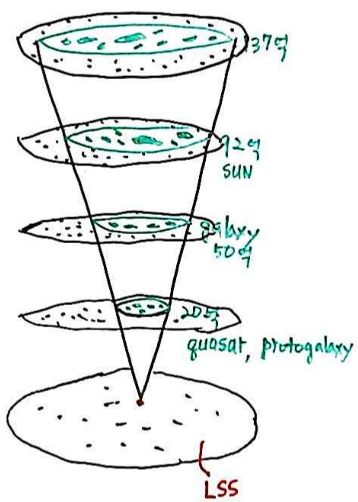
$$C_\ell(\eta_0) = \frac{\pi \Omega_{m0}}{2 D^2(a_0)} \delta_H^2 \frac{1}{\ell(\ell+1)}$$

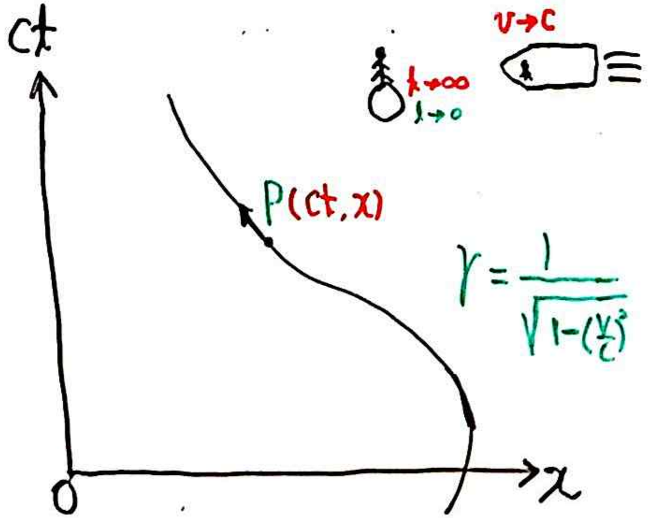

horizon
LSS η_*
ν, γ, DM, M


30km/s
COBE 8km/s
220km/s
$l = 2$
M31
MC
local group
680km/s
$\langle T \rangle = 2.725$ °K
l_0
Virgo
cluster
dipole
680 km/s
l_1
red shift
blue "
Hydra-Centaurus
super cluster
2.725°K
137 η_0




1°
½°
⅓°
¼°
⅕°
l


137억
92억
SUN
galaxy
50억
20억
quasar, protogalaxy
LSS



$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$t = \frac{\tau}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma\tau \qquad m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$dt = \gamma d\tau$$

$$\frac{dt}{d\tau} = \gamma$$

$$l = l_0\sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$\vec{x} = (ct,\ x) \qquad \vec{x}\cdot\vec{x} = (ct)^2 - x^2 \qquad \vec{u} = (c\gamma,\ v\gamma)$$

$$\vec{u} = \frac{d\vec{x}}{d\tau} = \left(\frac{d(ct)}{d\tau},\ \frac{dx}{dt}\frac{dt}{d\tau}\right) = \left(c\frac{dt}{d\tau},\ v\frac{dt}{d\tau}\right) = (c\gamma,\ v\gamma)$$

$$|\vec{u}| = \sqrt{\vec{u}\cdot\vec{u}} = \sqrt{(c\gamma)^2-(v\gamma)^2} = \gamma\sqrt{c^2-v^2} = \frac{\sqrt{c^2-v^2}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{c\sqrt{c^2-v^2}}{\sqrt{c^2-v^2}} = c$$

$$\vec{p} = m\vec{u} = (m_0 c\gamma,\ m_0 v\gamma) \qquad m_0\gamma = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = m \qquad \vec{p}\cdot\vec{p} = m\vec{u}\cdot m\vec{u} = m^2\vec{u}\cdot\vec{u}$$

$$= (mc,\ mv) \qquad\qquad = m^2c^2$$

$$= \left(\frac{\overset{=E}{mc^2}}{c},\ mv\right) = \left(\frac{E}{c},\ mv\right) = \left(\frac{E}{c},\ p\right) \qquad \vec{p}\cdot\vec{p} = \left(\frac{E}{c}\right)^2 - p^2 = m^2c^2$$

$$E^2 = c^2p^2 + m^2c^4 \qquad E^2 = (cp)^2 + (mc^2)^2 \qquad E^2 - c^2p^2 = m^2c^4$$

$$m=0 \qquad E^2 = (cp)^2 \qquad E = cp \qquad p = \frac{E}{c} = \frac{h\nu}{c}$$