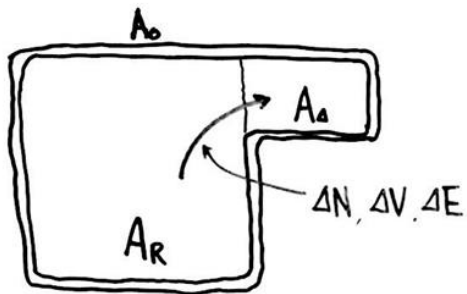


$$P_i \propto \Omega_i \rightarrow \Omega_i = e^{\frac{S_i}{k}} \rightarrow \bar{n} \rightarrow dN \rightarrow dE \rightarrow dU$$

$$S \equiv k \ln \Omega$$



$$\Omega_0 = \Omega_R \Omega_d = \Omega_R = e^{\frac{S_R}{k}}$$

$$U = Q - W + \mu N$$

$$\Delta U = T \Delta S - P \Delta V + \mu \Delta N$$

$$\Omega_0 = c e^{\frac{S_R}{k}}$$

$$S_R = S_R^0 - \Delta S$$

$$\Delta S = \frac{1}{T} (\Delta U + P \Delta V - \mu \Delta N)$$

$$\Omega_0 = c e^{\frac{1}{k} [S_R^0 - \frac{1}{T} (\Delta U + P \Delta V - \mu \Delta N)]} = c e^{-\frac{1}{kT} (\Delta U + P \Delta V - \mu \Delta N)}$$

$$\Omega_0 = c e^{-\beta (\Delta E + P \Delta V - \mu \Delta N)}$$



$$P_n = c e^{-\beta n (\epsilon_s - \mu)}$$

$$\bar{n} = \sum_n P_n n \quad \beta (\epsilon_s - \mu) \equiv x$$

$$\sum_n P_n = 1 \quad \sum_n c e^{-\beta n (\epsilon_s - \mu)} = 1$$

$$C = \frac{1}{\sum_n e^{-nx}} \quad P_n = \frac{e^{-nx}}{\sum_n e^{-nx}}$$

$$\bar{n} = -$$

$$\bar{n} = -$$

$$P_s = c e^{-\beta \epsilon_s}$$

$$P_n = c e^{-\beta n (\epsilon_s - \mu)}$$

$$\bar{n} = \frac{\sum_n e^{-n\chi}}{\sum_n e^{-n\chi}}$$

$\equiv \chi$

$$\bar{n} = -\frac{\partial}{\partial \chi} (\ln \sum_n e^{-n\chi})$$

1

$$\sum_{n=0,1} e^{-n\chi} = 1 + e^{-\chi} \quad \text{FD} \quad \text{spin} = \frac{1}{2} \quad \text{Fermion } e^-$$

$$\frac{e^{-n\chi}}{\sum_n e^{-n\chi}}$$

$$\sum_{n=0}^{\infty} e^{-n\chi} = 1 + e^{-\chi} + e^{-2\chi} + \dots = \frac{1}{1 - e^{-\chi}} \quad \text{BE} \quad \text{spin} = 1 \quad \text{boson } \gamma$$

$$\bar{n} = -\frac{\partial}{\partial \chi} [\ln(1 + e^{-\chi})] = \frac{e^{-\chi}}{1 + e^{-\chi}} = \frac{1}{e^{\chi} + 1}$$

$$\bar{n} = -\frac{\partial}{\partial \chi} [\ln(1 - e^{-\chi})^{-1}] = \frac{e^{-\chi}}{1 - e^{-\chi}} = \frac{1}{e^{\chi} - 1}$$

$$dN = \left(\frac{4\pi V p^2}{h^3} \right) \bar{n}$$

$$dN = \left(\frac{d^3r d^3p}{h^3} \right) \bar{n}$$

$$= \frac{V 4\pi p^2 dp}{h^3} \bar{n}$$

$$E^2 = (pc)^2 + (mc^2)^2$$

$$E = pc \quad p = \frac{E}{c}$$

$$dN = \frac{4\pi V p^2 dp}{h^3} \bar{n}$$

$$dN = \frac{8\pi V (\frac{E}{c})^2 \frac{dE}{c}}{h^3} \bar{n}$$

$$dN = \frac{8\pi V E^2}{c^3 h^3} dE \bar{n}$$

$$dE = \varepsilon dN$$

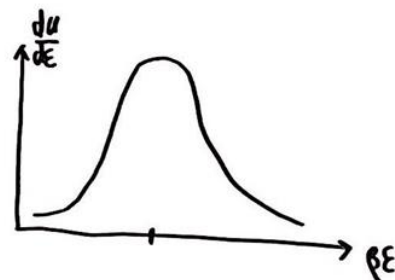
$$E = h\nu$$

$$dE = \frac{8\pi V E^3}{c^3 h^3} d\varepsilon \bar{n} \quad du = \frac{8}{c^3}$$

$$du = \frac{dE}{V} = \frac{8\pi E^3}{c^3 h^3} d\varepsilon \bar{n} \quad du = \frac{1}{c^3}$$

$$du = \frac{8\pi E^3}{c^3 h^3} \frac{1}{e^{\beta(E-\mu)} - 1} d\varepsilon$$

$$\frac{du}{d\varepsilon} = \left(\frac{8\pi}{c^3 h^3} \right) \frac{E^3}{e^{\beta E} - 1} \quad U$$



$U =$

$U =$

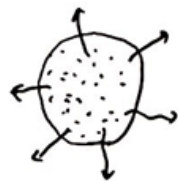
$$dE = \epsilon dN$$

$$E = h\nu$$

$$\epsilon = \frac{h}{2\pi} \cdot 2\pi\nu = \hbar\omega \quad \omega = \frac{\epsilon}{\hbar}$$

$$\hbar = \frac{h}{2\pi}$$

$$h = 2\pi\hbar$$



$$dE = \frac{8\pi V \epsilon^3}{c^3 h^3} d\epsilon \cdot \bar{n}$$

$$du = \frac{8\pi (\hbar\omega)^3 \hbar d\omega}{c^3 h^3} \frac{1}{e^{\beta\hbar\omega} - 1}$$

$$\text{선속} = \left(\frac{1}{2}U\right) \left(\frac{1}{2}c\right)$$

$$= \frac{c}{4}U$$

$$du = \frac{dE}{V} = \frac{8\pi \epsilon^3}{c^3 h^3} d\epsilon \cdot \bar{n}$$

$$du = \frac{8\pi (\hbar\omega)^3 \hbar}{c^3 (2\pi\hbar)^3} \frac{d\omega}{e^{\beta\hbar\omega} - 1}$$

$$\beta\hbar\omega \equiv x$$

$$\omega = \frac{x}{\beta\hbar}$$

$$\beta = \frac{1}{kT}$$

$$\text{선속} = \frac{c}{4}U = \frac{c}{4} \frac{8\pi^5 K^4}{15 c^3 h^3} T^4$$

$$= \frac{2\pi^5 K^4}{15 c^2 h^3} T^4 = a T^4$$

Stepan's law 1876

$$a = 5.67 \times 10^{-8} \text{ J/m}^2 \cdot \text{s} \cdot \text{K}^4$$

$$du = \frac{8\pi \epsilon^3}{c^3 h^3} \frac{1}{e^{\beta(\epsilon - \mu)} - 1} d\epsilon$$

$$du = \frac{\hbar}{c^3 \pi^2} \frac{\omega^3}{e^{\beta\hbar\omega} - 1} d\omega$$

$$\frac{du}{d\epsilon} = \left(\frac{8\pi}{c^3 h^3} \right) \frac{\epsilon^3}{e^{\beta\epsilon} - 1}$$

$$U = \int du = \frac{\hbar}{\pi^2 c^3} \int \frac{\left(\frac{x}{\beta\hbar}\right)^3 \frac{dx}{\beta\hbar}}{e^x - 1}$$

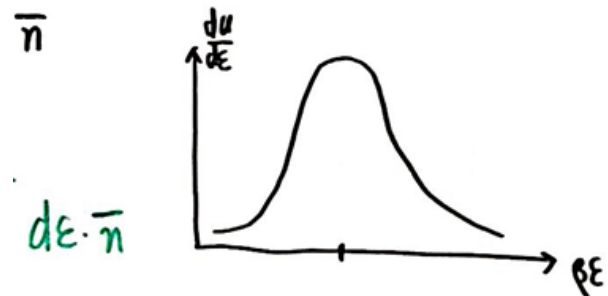
$$U = \frac{\hbar}{\pi^2 c^3 (\beta\hbar)^4} \int_0^\infty \frac{x^3}{e^x - 1} dx$$

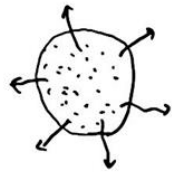
$$\rightarrow 7.56 \times 10^{-16} \text{ J/m}^3 \cdot \text{K}^4$$

$$U = \frac{(kT)^4}{\pi^2 c^3 \left(\frac{h}{2\pi}\right)^3} \frac{\pi^4}{15} = \frac{8\pi^5 K^4}{15 c^3 h^3} T^4 = a T^4$$

$$\text{선속}_{\text{SUN}} = a T^4 = 5.67 \times 10^{-8} \text{ J/m}^2 \cdot \text{s} \cdot \text{K}^4 (5800 \text{ K})^4$$

$$= 6.2 \times 10^7 \text{ J/m}^2 \cdot \text{s} = 6.2 \times 10^4 \text{ kW/m}^2$$





$$\dot{U} = \left(\frac{1}{2}U\right)\left(\frac{1}{2}c\right)$$

$$= \frac{c}{4}U$$

$\frac{x}{\beta h}$

$$\dot{U} = \frac{c}{4}U = \frac{c}{4} \frac{8\pi^5 K^4}{15c^3 h^3} T^4$$

$$= \frac{2\pi^5 K^4}{15c^2 h^3} T^4 = aT^4$$

Stepan's law 1876

$$a = 5.67 \times 10^{-8} \text{ J/m}^2 \cdot \text{s} \cdot \text{K}^4$$

$$6 \times 10^{-16} \text{ J/m}^2 \cdot \text{s} \cdot \text{K}^4$$

$$a = aT^4$$

$$\dot{U}_{\text{SUN}} = aT^4 = 5.67 \times 10^{-8} \text{ J/m}^2 \cdot \text{s} \cdot \text{K}^4 (5800 \text{ K})^4$$

$$= 6.2 \times 10^7 \text{ J/m}^2 \cdot \text{s} = 6.2 \times 10^4 \text{ kW/m}^2$$

$$T = \left(\frac{3c^2}{32\pi^5 a}\right)^{\frac{1}{4}} \frac{1}{\sqrt{f}} = 1.5 \times 10^{10} \frac{1}{\sqrt{f}}$$

$$U = Q - W + \mu N$$

$$\Delta U = T \Delta S - P \Delta V + \mu \Delta N$$

$$du = T ds - p dv + \mu dn$$

$$du + p dv = 0$$

$$dE + p dv = 0$$

$$V = \frac{4\pi}{3} r^3$$

$$E = mc^2 = \frac{4\pi}{3} r^3 \rho c^2$$

$$dE = \frac{4\pi}{3} r^3 \dot{\rho} c^2 + \frac{4\pi}{3} r^3 \rho \dot{c}^2 + \frac{4\pi}{3} \dot{r}^3 \rho c^2 = 0$$

$$\dot{\rho} c^2 + \frac{r}{3} \dot{\rho} c^2 + \dot{r} \rho c^2 = 0$$

$$\dot{\rho} + 3 \frac{\dot{r}}{r} \rho + 3 \frac{\dot{r}}{r} \frac{P}{c^2} = 0$$

$$\frac{\dot{\rho}}{\rho} + 3 \frac{\dot{r}}{r} + 3 \frac{\dot{r}}{r} \frac{P}{\rho c^2} = 0$$

$$\frac{\dot{\rho}}{\rho} = -3 \frac{\dot{r}}{r} \left(1 + \frac{P}{\rho c^2}\right)$$

$$P = w \rho c^2 \quad w = \frac{1}{3} \quad \frac{\dot{r}}{r} = H \quad \dot{\rho} = \frac{d\rho}{dt}$$

$$P = \frac{1}{3} a T^4 \quad w = 0 \quad H^2 = \frac{8\pi G}{3} \rho \quad w = -1$$

$$\frac{\dot{\rho}}{\rho} = -3H \left(1 + \frac{\frac{1}{3} \rho c^2}{\rho c^2}\right) = -3H \left(1 + \frac{1}{3}\right) = -4H$$

$$\dot{\rho} = -4H\rho = -4 \left(\frac{8\pi G}{3} \rho\right)^{\frac{1}{2}} \rho \quad \rho = \frac{a}{c^2} T^4$$

$$\dot{\rho} \rho^{-\frac{5}{2}} = -\left(\frac{128\pi G}{3}\right)^{\frac{1}{2}}$$

$$\int \rho^{-\frac{5}{2}} d\rho = -\int \left(\frac{128\pi G}{3}\right)^{\frac{1}{2}} dt$$

$$\frac{\rho^{-\frac{1}{2}}}{-\frac{5}{2}+1} = -\left(\frac{128\pi G}{3}\right)^{\frac{1}{2}} t \quad \rho^{-\frac{1}{2}} = \left(\frac{32\pi G}{3}\right)^{\frac{1}{2}} t$$

$$\rho^{-\frac{1}{2}} = \frac{1}{\rho^{\frac{1}{2}}} = \frac{1}{\left(\frac{a}{c^2} T^4\right)^{\frac{1}{2}}} = \left(\frac{32\pi G}{3}\right)^{\frac{1}{2}} t$$

$$T^2 = \frac{1}{\left(\frac{a}{c^2}\right)^{\frac{1}{2}} \left(\frac{32\pi G}{3}\right)^{\frac{1}{2}} t} = \left(\frac{3c^2}{32\pi G a}\right)^{\frac{1}{2}} \frac{1}{t}$$

$$\int \frac{dt}{t} = \ln t = \ln t_0 + \ln \frac{t}{t_0}$$

