$$-(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{dw}{\sqrt{1-kw^2}}\right)^2 + (w\,d\theta)^2 + (w\sin\theta\,d\phi)^2\right]$$

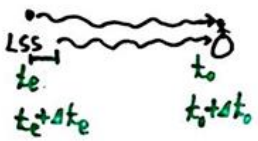
FRW metric

light-like $ds=0$ $\quad d\theta=d\phi=0$

$$c\,dt = R(t)\frac{dw}{\sqrt{1-kw^2}}$$

$$\frac{c\,dt}{R(t)} = \frac{dw}{\sqrt{1-kw^2}}$$

$$\int_{t_e}^{t_0}\frac{c\,dt}{R(t)} = \int_{w_0 \to 0}^{w_e}\frac{dw}{\sqrt{1-kw^2}}$$



$$\int_{t_e+\Delta t_e}^{t_0+\Delta t_0}\frac{c\,dt}{R(t)} = \int_0^{w_e}\frac{dw}{\sqrt{1-kw^2}}$$

$$\int_{t_e+\Delta t_e}^{t_0+\Delta t_0}\frac{c\,dt}{R(t)} - \int_{t_e}^{t_0}\frac{c\,dt}{R(t)} = 0$$

$$\int_{t_e+\Delta t_e}^{t_e} " + \int_{\underset{\sim}{t_e}}^{t_0} " + \int_{t_0}^{t_0+\Delta t_0} " - \int_{\underset{\sim}{t_e}}^{t_0} " = 0$$

$$\int_{t_e+\Delta t_e}^{t_e}\frac{c\,dt}{R(t)} + \int_{t_0}^{t_0+\Delta t_0}\frac{c\,dt}{R(t)} = 0$$

$$\frac{[t]_{t_e+\Delta t_e}^{t_e}}{R(t_e)} + \frac{[t]_{t_0}^{t_0+\Delta t_0}}{R(t_0)} = 0$$

$$\frac{t_e-(t_e+\Delta t_e)}{R(t_e)} + \frac{(t_0+\Delta t_0)-t_0}{R(t_0)} = 0$$

$$\frac{-\Delta t_e}{R(t_e)} + \frac{\Delta t_0}{R(t_0)} = 0$$

$$\frac{\Delta t_e}{R(t_e)} = \frac{\Delta t_0}{R(t_0)} = \Delta t_0$$

$$\Delta t_0 = \frac{1}{R(t_e)}\Delta t_e$$

cosmic time expension

$\lambda = c\,\Delta t \qquad \lambda_0 = c\,\Delta t_0$

$$\frac{1}{R(t_e)} = \frac{\Delta t_0}{\Delta t_e} = \frac{\left(\frac{\lambda_0}{c}\right)}{\left(\frac{\lambda_e}{c}\right)} = \frac{\lambda_0}{\lambda_e}$$

$$Z \equiv \frac{\lambda_0-\lambda_e}{\lambda_e} \qquad z = \frac{\lambda_0}{\lambda_e} - 1$$

$$\frac{\lambda_0}{\lambda_e} = Z+1 = \frac{1}{R(t_e)}$$

$$d_p(t) = R(t)\int_{t_e}^{t_0}\frac{c\,dt}{R(t)}$$

$$d_h(t) = R(t)\int_0^{t}\frac{c\,dt}{R(t)}$$

RDU $\quad R(t) = C' t^{\frac{1}{2}}$

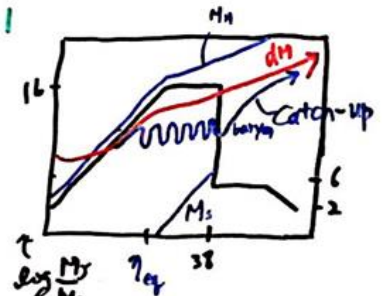
$$d_h(t) = \underset{\sim}{C'} t^{\frac{1}{2}}\int_0^t \frac{c\,dt}{\underset{\sim}{C'} t^{\frac{1}{2}}}$$

$$= c t^{\frac{1}{2}}\int_0^t t^{-\frac{1}{2}}dt$$

$$= c t^{\frac{1}{2}}\frac{1}{-\frac{1}{2}+1}t^{\frac{1}{2}} = 2ct$$

MDU $\quad R(t) = C' t^{\frac{2}{3}}$

$$d_h(t) = 3ct$$



$d_p(t_0$

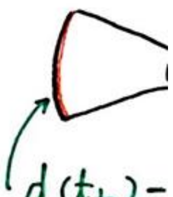
$d_p(t_0) =$

$=$

$= c$

$= c$

$= 3c$



$d_s(t_{dec}) =$

$$M_J = \rho_{bary}\left(\frac{4\pi}{3}\lambda_J\right)^3$$

$$d_p(t_0) = R(t_0)\int_{t_{dec}}^{t_0}\frac{c\,dt}{R(t)}$$

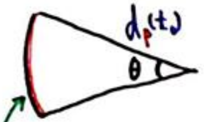
$$d_p(t_0) = C' t_0^{\frac{2}{3}}\int_{t_{dec}}^{t_0}\frac{c\,dt}{C' t^{\frac{2}{3}}}$$

$$= c t_0^{\frac{2}{3}}\int_{t_{dec}}^{t_0} t^{-\frac{2}{3}}dt$$

$$= c t_0^{\frac{2}{3}}\left.\frac{t^{\frac{1}{3}}}{-\frac{2}{3}+1}\right|_{t_{dec}}^{t_0}$$

ct $$= c t_0^{\frac{2}{3}}\cdot 3\cdot\left[(t_0)^{\frac{1}{3}}-(t_{dec})^{\frac{1}{3}}\right]$$

$C' t^{\frac{2}{3}}$ $$= 3ct_0\left[1-\left(\frac{t_{dec}}{t_0}\right)^{\frac{1}{3}}\right]$$



up

$$d_s(t_{dec}) = \frac{d_h}{\sqrt{3}} = \frac{3ct}{\sqrt{3}} = \sqrt{3}\,c\,t_{dec}$$

$$= 201\ \text{Kpc}$$

$$d_p(t_0)\,\theta = d_s(t_{dec})(1+z_{dec})$$

$$3ct_0\left[1-\left(\frac{t_{dec}}{t_0}\right)^{\frac{1}{3}}\right]\theta = d_s(t_{dec})(1+1089)$$

$t_{dec} \to 38$, $t_0 \to 137$, $d_s(t_{dec}) \to 201$ Kpc

$$\theta = 1.03^\circ \qquad \ell = \frac{180^\circ}{1.03^\circ} = 175$$

~ 200

flat universe

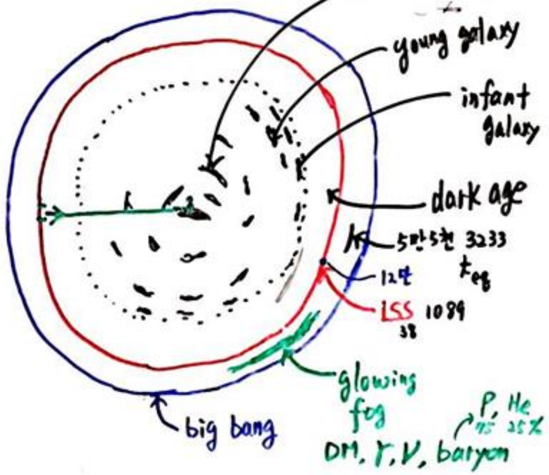
$$\ell(\ell+1)C_\ell^{LS} = \frac{\pi\,\Omega_{m0}^2}{2D^2(a_0)}\delta_H^2$$

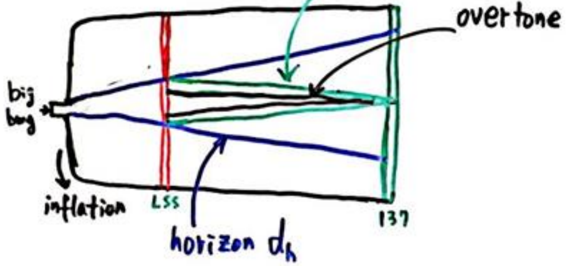
LS → $\ell < 20$

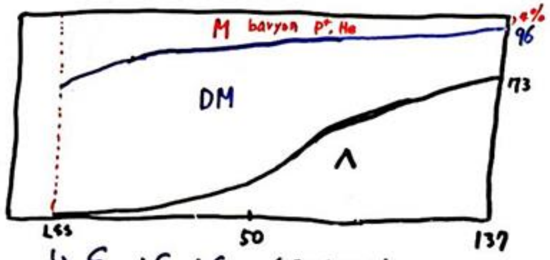
F^3 $$M_J = 1\times10^{+5} M_\odot$$

$\sim$ → $7\times10^{18} M_\odot$

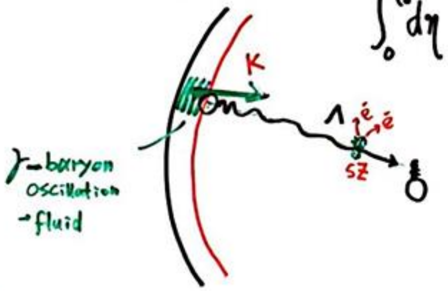
→ at 38만







→ $\varepsilon_{DM} : \varepsilon_\gamma : \varepsilon_b = 6.4 : 1.4 : 1$



$$\int_0^{\eta_0} d\eta\, e^{-\tau}(\dot{\Psi}-\dot{\Phi})\,j_\ell(k(\eta_0-\eta)) \to \text{ISW}$$

$$ds = a^2(t)\left[-(1+2\Psi) - (1+2\Phi)\delta_{ij}\,dx^i dx^j\right]$$

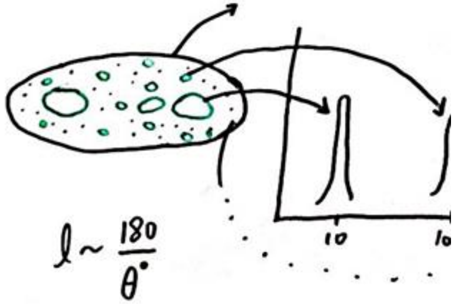
$$\text{SW} \begin{cases} \text{SW} & \dfrac{\delta T}{T} = \dfrac{1}{3}\dfrac{\delta\Phi}{c^2} \\ \text{ISW} & \begin{cases} \text{early remanent } \gamma \text{ at LSS} \\ \text{late } \Lambda \end{cases} \end{cases}$$

$\ell(\ell+$

$$\theta_\ell(\eta_0) = [\theta_0(\eta_* ...$$

$$+3\theta_1[j_{\ell-1}(k(...$$

$$+\int_0^{\eta_0}d\eta\, e^{-\tau}[...$$



$$\ell \sim \frac{180}{\theta^\circ}$$

xy

ant
galaxy

k age
3233
t_{eq}
19

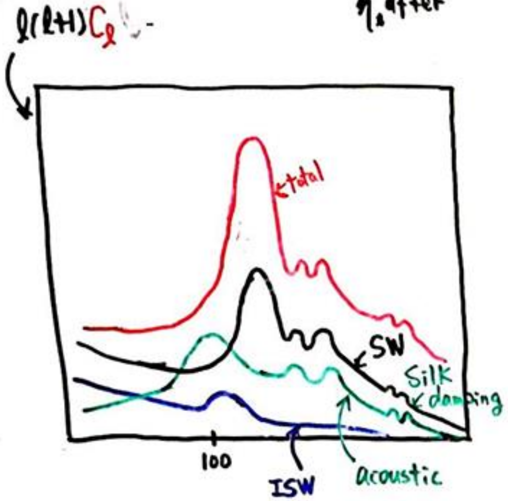
P, He
75 25%
baryon

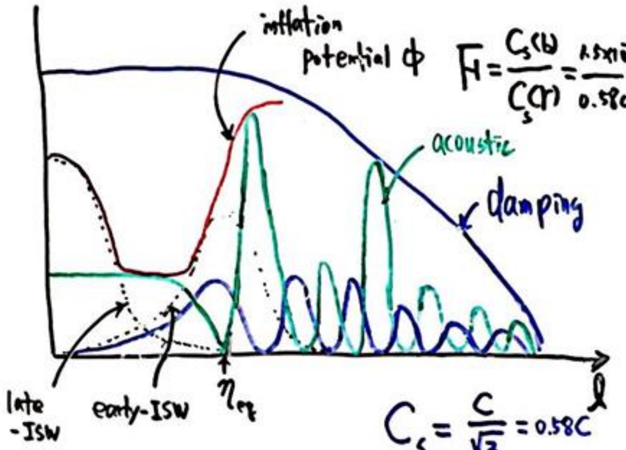
0%
96
73
137

55

$$C_s(\text{baryon}) = \sqrt{\frac{P}{\rho}}\,c = \left(\frac{kT}{mc^2}\right)^{\frac{1}{2}} c = \left(\frac{0.26\,\text{eV}}{1140\times10^6\,\text{eV}}\right)^{\frac{1}{2}} C = 1.5\times10^{-5}\,C$$
η_* after
$$= 5\times10^3\ \text{m/s}$$

$$F \to \frac{\ddot R}{R} \to M \to \delta(t) \to \lambda_J \to M_J$$





$$C_s = \frac{c}{\sqrt3} = 0.58C$$

$$F = \frac{C_s(b)}{C_s(r)} = \frac{1.5\times10^{-5}c}{0.58c} \qquad F = \frac{-GMm}{R^2} \qquad a = \frac{d^2R}{dt^2} = \ddot R \qquad a = \frac{F}{m} \qquad \ddot R = -\frac{G\,\Delta M}{R^2}$$

$$\ddot R = -\frac{G}{R^2}\Big(\underbrace{\frac{4\pi}{3}R^3\bar\rho\,\delta}_{\Delta M}\Big) \qquad \frac{\ddot R}{R} = -\frac{4\pi G}{3}\bar\rho\,\delta$$

$$M = \frac{4\pi}{3}[1+\delta(t)]R^3\bar\rho \qquad R^3 = \frac{3M}{4\pi\bar\rho}(1+\delta)^{-1}$$

$$R = \underbrace{\left(\frac{3M}{4\pi\bar\rho}\right)^{\frac13}}_{R_0}(1+\delta)^{-\frac13} = R_0(1+\delta)^{-\frac13} \qquad \delta(t) \ll 1$$

$$R \simeq R_0\left(1-\tfrac13\delta\right) = R_0 - \tfrac13 R_0\delta$$

$$\ddot R = -\tfrac13 R_0\ddot\delta \qquad \frac{\ddot R}{R_0} = -\frac13\ddot\delta = -\frac{4\pi G}{3}\bar\rho\,\delta \qquad \delta(t)$$

$$\ddot\delta = 4\pi G\bar\rho\,\delta \qquad \ddot\delta - \underbrace{4\pi G\bar\rho}_{K^2}\,\delta = 0$$

$$\ddot\delta - K^2\delta = 0 \qquad \frac{d^2\delta}{dt^2} - K^2\delta = 0 \qquad \left(\frac{d}{dt}-K\right)\left(\frac{d}{dt}+K\right)\delta = 0$$

$$\frac{d\delta}{dt} - K\delta = 0 \qquad \frac{d\delta}{\delta} = K\,dt \qquad \int\frac{d\delta}{\delta} = K\int dt$$

$$\ln\delta = Kt \qquad e^{\ln\delta} = e^{Kt} \qquad \delta = e^{Kt}$$

$l=0$

$$\Theta_l(\eta_0) = [\Theta_0(\eta_*) + \Psi(\eta_*)]\,j_l(k(\eta_0-\eta_*))$$

$l=1$

$$+\,3\Theta_1\left[j_{l-1}(k(\eta_0-\eta_*)) - \frac{l+1}{k(\eta_0-\eta_*)}\,j_l(k(\eta_0-\eta_*))\right]$$

ISW

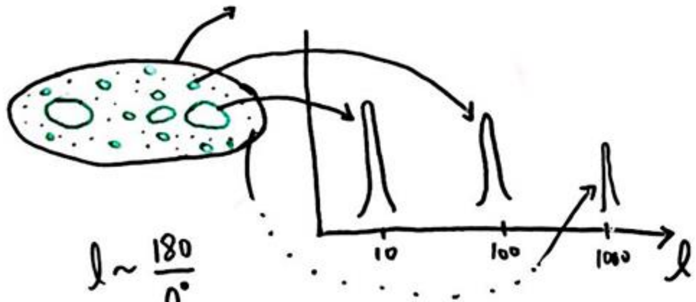
$$+\int_0^{\eta_0} d\eta\, e^{-\tau}\left[\dot\Psi(\eta) - \dot\Phi(\eta)\right] j_l(k(\eta_0-\eta))$$

$$\delta(t) = e^{Kt} = e^{\frac{t}{t_d}} \qquad t_d = \frac1K = \frac{1}{(4\pi G\bar\rho)^{\frac12}}$$

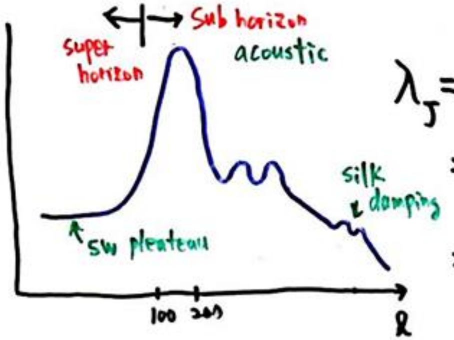
$$H^2 = \frac{8\pi G}{3}\rho \qquad \varepsilon = \rho c^2 \qquad \bar\rho = \frac{\bar\varepsilon}{c^2} \qquad H^2 = \frac{8\pi G\bar\varepsilon}{3c^2}$$

$$\frac1H = \left(\frac{3c^2}{8\pi G\varepsilon}\right)^{\frac12} \qquad t_d = \left(\frac{c^2}{4\pi G\varepsilon}\right)^{\frac12}$$

$$\frac1H = \left(\frac32\right)^{\frac12} t_d \qquad t_d = \left(\frac23\right)^{\frac12}\frac1H$$



$$l \sim \frac{180}{\theta^\circ}$$



$$\lambda_J = 2\pi C_s t_d$$
$$= 2\pi\frac{c}{\sqrt3}\left(\frac23\right)^{\frac12}\frac1H$$
$$= \frac{2\sqrt2\pi}{3}\frac{c}{H} = 3\frac{c}{H(\eta_*)}$$
$\hookrightarrow$ 0.2 Mpc