1강

1. $P_i \propto \Omega_i \Rightarrow \Omega_i = e^{\frac{S_i}{k}} \Rightarrow \bar{n} \Rightarrow dN \Rightarrow dE \Rightarrow du$

2. $S \equiv k \ln \Omega$

3. 열역학 제 1 법칙

$$\Delta U = T\Delta S - P\Delta V + \mu \Delta N$$

4. $$\begin{cases} P_n = ce^{-\beta n(\varepsilon_s - \mu)} \\ \Omega_0 = ce^{-\beta(\Delta E + P\Delta V - \mu \Delta N)} \end{cases} \quad \beta = \frac{1}{KT}$$

5. $\bar{n} = -\frac{\partial}{\partial x}\left(\ln \sum_n e^{-nx}\right) \Rightarrow \begin{cases} \bar{n} = \frac{1}{e^x + 1} & \text{(페르미온)} \\ \bar{n} = \frac{1}{e^x - 1} & \text{(보존)} \end{cases}$

$x = \beta(\varepsilon_s - \mu)$

6. $dN = \frac{8\pi V \varepsilon^2}{c^3 h^3} d\varepsilon \cdot \bar{n}$

$dE = \varepsilon \cdot dN$

7. $du = \frac{dE}{V} = \frac{8\pi \varepsilon^3}{c^3 h^3} d\varepsilon \cdot \bar{n}$

8. $\frac{du}{d\varepsilon} = \left(\frac{8\pi}{c^3 h^3}\right) \frac{\varepsilon^3}{e^{\beta\varepsilon} - 1}$ 막스플랑크 흑체복사곡선

9. $u = \sigma T^4$ 스테판 볼츠만 공식 $\sigma = 5.67 \times 10^{-8} J s^{-1} m^{-2} K^{-4}$

10. $p = \omega \rho c^2$ $p = \frac{1}{3} a T^4$ $\frac{\dot{r}}{r} = H$ $T = 1.5 \times 10^{10} \frac{1}{\sqrt{t}}$

2강

1.

$$\frac{du}{d\varepsilon} = \frac{8\pi}{c^3h^3}\frac{\varepsilon^3}{e^{\beta\varepsilon}-1}$$

$E = h\nu$

$$\frac{du}{d\lambda} = 8\pi ch\frac{\lambda^{-5}}{e^{\beta\hbar c/\lambda}-1}$$

$\omega = 2\pi\nu$

$$\frac{du}{d\omega} = \frac{\hbar}{\pi^2c^3}\frac{\omega^3}{e^{\beta\hbar\omega}-1}$$

$\hbar = \frac{h}{2\pi}$

$$\frac{du}{d\lambda} = 0$$

$\lambda_{max}T = 0.289\ cm\,°K$ (빈의 변위법칙)

2. Ideal gas

$$PV = NkT$$

$$u = \frac{3}{2}NkT$$

$$kT = \frac{1}{3}mv^2$$

$$p = w\rho c^2 \qquad w = \frac{v^2}{3c^2}$$

볼츠만상수 $k = 8.617\times10^{-5}\,eV/K$

RDU $w = \frac{1}{3}$

MDU $w = 0$

ΛDU $w = -1$

3. 유체방정식

$$\frac{\dot\rho}{\rho} = -3(1+w)\frac{\dot R}{R}$$

4. 프리드만 방정식

$$\left(\frac{\dot R}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$$

5.

$$R(t) = \left(\frac{\Omega_{m0}}{\Omega_{\Lambda0}}\right)^{\frac{1}{3}}\sinh^{\frac{2}{3}}\left(\frac{3}{2}H_0t\sqrt{\Omega_{\Lambda0}}\right)$$

$H_0 = 3.24\times10^{-18}h/sec.$ $\quad h = 0.71$

3강

1. 특수상대성 이론

길이 $l = l_0\sqrt{1-\left(\frac{v}{c}\right)^2}$

질량 $m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ $\quad \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\beta^2}}$. 로렌츠 인자

시간 $t = \frac{\tau}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma\tau$ $\quad \tau$. 고유시간

위치벡터 $\vec{x} = (ct, x)$

4차원 속도벡터 $\vec{u} = (c\gamma, v\gamma)$

$|\vec{u}| = \gamma\sqrt{c^2-v^2} = c$

4차원모멘텀벡터 $\vec{p} = m_0\vec{u}$ $\quad m_0$ 정지질량

$$E^2 = (cp)^2 + (mc^2)^2$$

4강

1. $f \rightarrow \frac{df}{d\eta} \rightarrow c[f] \rightarrow \theta(\eta_0) \rightarrow S \rightarrow \frac{\partial}{\partial\eta}(\theta e^{ik\mu\eta-\tau})$
$\rightarrow \bar{S} \rightarrow j_l \rightarrow \theta_l(\eta_0)$

2. Boltzmann Equation

$$\frac{df}{dt} = c[f] \qquad \frac{df}{d\eta} = ac[f] \qquad c=h=k_b=1$$

$$c[f] = -p\frac{\partial f_0}{\partial p} n_e \sigma_T(\theta_0 - \theta + V_b \cdot \vec{n})$$

3. Geodesic Equation of γ (photon)

$$\frac{1}{P}\frac{dP}{d\eta} = -\mathcal{H} - \frac{\partial\Phi}{\partial\eta} - n^i\frac{\partial\Psi}{\partial x^i}$$

cf) 아인슈타인의 측지선방정식 . $\frac{du^\lambda}{ds} + \Gamma^\lambda_{\mu\nu} u^\mu u^\nu = 0$

4. FRW metric

$$ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1+2\Phi)\delta_{ij}dx^i dx^j]$$

$g_{00} = -(1+2\Psi)$ $\qquad g_{ij} = 1+2\Phi \qquad$ a. scale factor

$$g_{\mu\nu} = a^2(\eta)\,\mathrm{diag}[-1.1.1.1] \qquad g_{\mu\nu} = \frac{1}{g^{\mu\nu}}$$

5. 중력장 방정식

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{8\pi G}{c^4} T^{\mu\nu}$$

6. First-order Boltzmann Eq for γ in Fourier space

$$\dot{\Theta} + i\mu k \Theta + \dot{\Phi} + ik\mu\Psi = -\dot{\tau}(\Theta_0 - \Theta + \mu v_b)$$

7. $\mu = \frac{-1}{ik}\frac{\partial}{\partial\eta}$ $g \equiv -\dot{\tau} e^{-\tau}$ $\mu \equiv \frac{\vec{k}\cdot\vec{n}}{k}$

8. $\int_{-1}^{1} d\mu P_\ell(\mu) e^{i\mu x} = \frac{2 j_\ell(x)}{(-i)^\ell}$ $x = k(\eta - \eta_0)$

9. $$\Theta_\ell(\eta_0) = [\Theta_0(\eta_*) + \Psi(\eta_*)] j_\ell(k(\eta_0 - \eta_*)) + 3\Theta_1 \Big[j_{\ell-1}(k(\eta_0 - \eta_*)) - \frac{\ell+1}{k(\eta_0 - \eta_*)} j_\ell(k(\eta_0 - \eta_*)) \Big] + \int_0^{\eta_0} d\eta\, e^{-\tau}(\dot{\Psi} - \dot{\Phi}) j_\ell(k(\eta_0 - \eta))$$

10. $\mathcal{L} \to \ddot{\phi} \to \phi + \delta\phi \to \ddot{\delta\phi} \to \ddot{h} \to \delta\phi$
$\to \zeta \to \Psi \to P_\Psi \to \delta \to P(k)$

11. $C_\ell(\eta) = \frac{2}{\pi}\int_0^{\eta_0} dk\, k^2 P(k) \left|\frac{\Theta_\ell(k)}{\delta(k)}\right|^2$

12. Lagrangian

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu} - V(\phi)$$

13. Euler-Lagrange Eq

$$\frac{\partial}{\partial x^\mu}\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right) - \frac{\partial\mathcal{L}}{\partial\phi} = 0$$

14. $P(k) = \langle \delta \cdot \delta^* \rangle = 2\pi^2 \delta_H^2 \frac{k^n}{H_0^{n+3}} \left(\frac{D(a)}{D(a_0)}\right)^2 T^2(k)$

15. $\ell(\ell+1) C_\ell^{LS}(\eta) = \frac{\pi\, \Omega_{m0}^2}{2D^2(a_0)} \delta_H^2$